

Solution to HW 3

Find the differential cross section for the $\pi^+\pi^- \Rightarrow \pi^+\pi^-$ scattering in the first nontrivial order in perturbation theory.

The differential cross section for a general $2 \Rightarrow 2$ particle cross section is calculated in Appendix A. For our case

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{1}{64\pi^2 s} |T|^2 \quad (1)$$

where $s = (E_1 + E'_1)^2$. The relevant diagrams are shown in Fig. 1 below. The transition

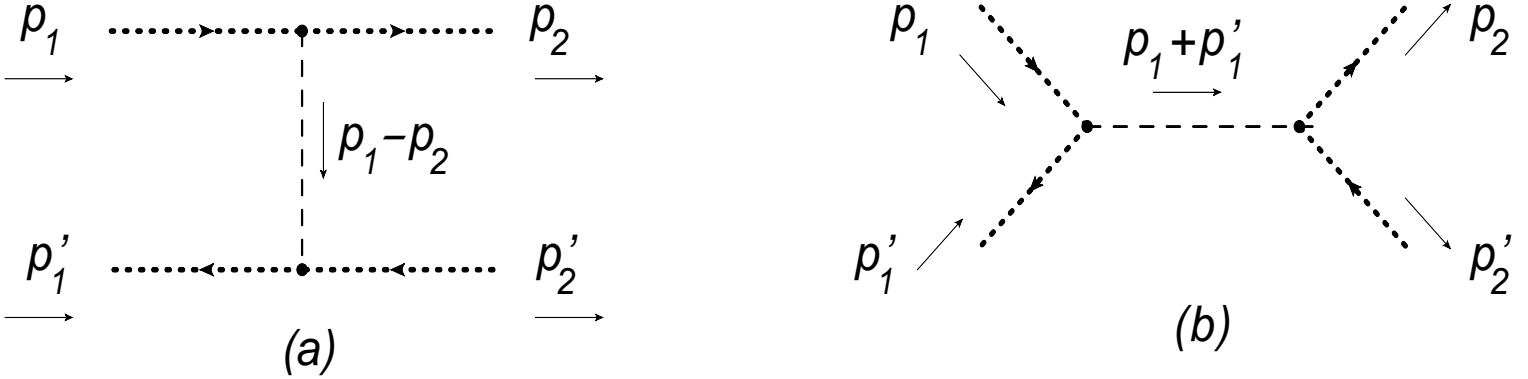


FIG. 1. Feynman diagrams for $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering.

matrix is the amputated reduced Green function:

$$\begin{aligned} T(p_2, p'_2; p_1, p'_1) &= \frac{e^2}{(p_1 - p_2)^2 + i\epsilon} (p_1 + p_2) \cdot (-p'_1 - p'_2) + \frac{e^2}{(p_1 + p'_1) + i\epsilon} (p_1 - p'_1) \cdot (p_2 - p'_2) \\ &= -\frac{e^2}{t} (p_1 + p_2) \cdot (p'_1 + p'_2) + \frac{e^2}{s} (p_1 - p'_1) \cdot (p_2 - p'_2). \end{aligned} \quad (2)$$

Using $p_1 \cdot p'_1 = p_2 \cdot p'_2 = \frac{s}{2} - m^2$, $p_1 \cdot p_2 = p'_1 \cdot p'_2 = m^2 - \frac{t}{2}$ and $p_1 \cdot p'_2 = p_2 \cdot p'_1 = m^2 - \frac{u}{2}$ we get

$$T(p_2, p'_2; p_1, p'_1) = -e^2 \left[\frac{s-u}{t} + \frac{t-u}{s} \right] \quad (3)$$

Note that the expressions for diagrams Fig. 1a and Fig1b are related by crossing symmetry $p_2 \leftrightarrow -p'_1 \Leftrightarrow s \leftrightarrow t$.

Using Eq. (4.121) from the lecture notes

$$s = 4E_1^2, \quad t = -4|\vec{p}_1|^2 \sin^2\left(\frac{\theta}{2}\right), \quad u = -4|\vec{p}_1|^2 \cos^2\left(\frac{\theta}{2}\right) \quad (4)$$

we get

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{e^4}{64\pi^2 s} \left[\frac{1 + \cos^2\frac{\theta}{2} - \frac{m^2}{|\vec{p}|^2}}{\sin^2\frac{\theta}{2}} - \frac{4|\vec{p}|^2}{s} \cos\theta \right] \quad (5)$$