

*Solution to HW 6*

The bispinor for an electron moving in Y direction with rest-frame spin in X direction can be obtained from Eq. (8.53):

$$u(p, s) = \frac{1}{\sqrt{2(p_0 + m)}} \begin{pmatrix} (m + p_\mu \sigma^\mu \bar{\sigma}_0) \kappa \\ (m + p_\mu \bar{\sigma}^\mu \sigma_0) \kappa \end{pmatrix} = \frac{1}{\sqrt{2(p_0 + m)}} \begin{pmatrix} (m + p_0 - \vec{p} \cdot \vec{\sigma}) \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi} \sin(\theta/2) \end{pmatrix} \\ (m + p_0 + \vec{p} \cdot \vec{\sigma}) \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi} \sin(\theta/2) \end{pmatrix} \end{pmatrix} \quad (1)$$

where  $\theta = \frac{\pi}{2}$ ,  $\phi = 0$ , and  $p = (p_0, 0, p, 0)$ . In explicit form  $\kappa = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and therefore

$$u(p, s) = \frac{1}{2\sqrt{p_0 + m}} \begin{pmatrix} (m + p_0 - \vec{p} \cdot \vec{\sigma}) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ (m + p_0 + \vec{p} \cdot \vec{\sigma}) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{pmatrix} = \frac{1}{2\sqrt{(p_0 + m)}} \begin{pmatrix} m + p_0 + ip \\ m + p_0 - ip \\ m + p_0 - ip \\ m + p_0 + ip \end{pmatrix} \quad (2)$$

so the wavefunction is

$$\psi^e(x) = \frac{1}{2\sqrt{(p_0 + m)}} \begin{pmatrix} m + p_0 + ip \\ m + p_0 - ip \\ m + p_0 - ip \\ m + p_0 + ip \end{pmatrix} \frac{1}{\sqrt{2p_0}} e^{-i(p_0 t - py)} \quad (3)$$

The bispinor for a positron moving in X direction with rest-frame spin in Y direction can be obtained from Eqs. (8.56) and (8.58):

$$\begin{aligned} \bar{v}(p, s) &= \frac{1}{\sqrt{2(p_0 + m)}} [\kappa^\dagger (m\sigma_0 + p_\mu \bar{\sigma}^\mu), \kappa^\dagger (-m - \bar{\sigma}_0 p_\mu \sigma^\mu)] = \\ &= \frac{1}{\sqrt{2(p_0 + m)}} \left[ \left( -e^{i\phi} \sin \frac{\theta}{2}, \cos \frac{\theta}{2} \right) (m + p_0 + \vec{p} \cdot \vec{\sigma}); \left( -e^{i\phi} \sin \frac{\theta}{2}, \cos \frac{\theta}{2} \right) (-m - p_0 + \vec{p} \cdot \vec{\sigma}) \right] \end{aligned} \quad (4)$$

where now  $\theta = \frac{\pi}{2}$ ,  $\phi = \frac{\pi}{2}$ , and  $p = (p_0, p, 0, 0)$ . In explicit form  $\kappa^\dagger = \frac{1}{\sqrt{2}}(-i, 1)$  (recall that  $\kappa^\dagger \sigma_y = -\kappa^\dagger$ ) and therefore

$$\begin{aligned} \bar{v}(p, s) &= \frac{1}{2\sqrt{p_0 + m}} [(-i, 1)(m + p_0 + p\sigma_x); (-i, 1)(-m - p_0 + p\sigma_x)] \\ &= \frac{1}{2\sqrt{p_0 + m}} [-i(m + p_0) + p, m + p_0 - ip_1, (m + p_0) + p, -(m + p_0) - ip] \end{aligned} \quad (5)$$

so the positron wavefunction takes the form:

$$\underline{\psi}^p(x) = \frac{1}{2\sqrt{p_0 + m}} [-i(m + p_0) + p, m + p_0 - ip_1, i(m + p_0) + p, -(m + p_0) - ip] \frac{e^{-i(p_0 t - px)}}{\sqrt{2p_0}} \quad (6)$$