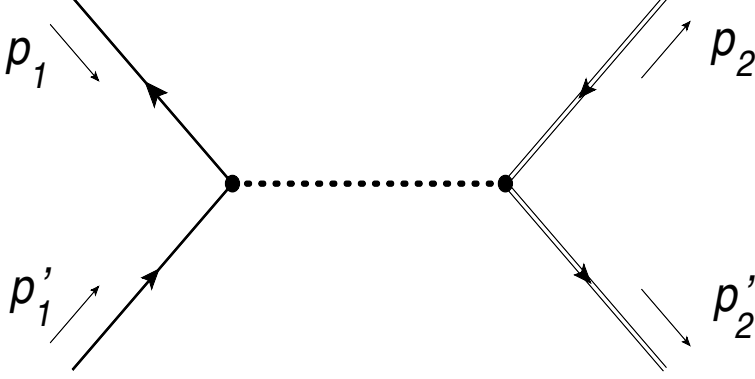


Solution to HW 7

The differential cross section of the unpolarized $e^+e^- \rightarrow \mu^+\mu^-$ annihilation is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_2|}{|\vec{p}_1|} \frac{1}{4} \sum_{\text{polarizations}} |T|^2 \quad (0.1)$$

where the transition matrix is determined by the diagram



$$T^{\lambda_1, \lambda'_1, \lambda_2, \lambda'_2} = \frac{e^2}{s} [\bar{U}^{\lambda_2}(p_2) \gamma^\mu V^{\lambda'_2}(p'_2)] [\bar{v}^{\lambda'_1}(p'_1) \gamma^\mu u^{\lambda_1}(p_1)] \quad (0.2)$$

We get

$$\begin{aligned} & \frac{1}{4e^4} \sum_{\lambda_1, \lambda'_1, \lambda_2, \lambda'_2} |T^{\lambda_1, \lambda'_1, \lambda_2, \lambda'_2}|^2 \\ &= \frac{1}{4s^2} [\bar{U}^{\lambda_2}(p_2) \gamma_\mu V^{\lambda'_2}(p'_2)] [\bar{v}^{\lambda'_1}(p'_1) \gamma^\mu u^{\lambda_1}(p_1)] [\bar{V}^{\lambda'_2}(p'_2) \gamma^\nu U^{\lambda_2}(p_2)] [\bar{u}^{\lambda_1}(p_1) \gamma_\nu v^{\lambda'_1}(p'_1)] \\ &= \frac{1}{4s^2} \text{Tr}\{(\not{p}_2 + M) \gamma^\mu (\not{p}'_2 - M) \gamma^\nu\} \text{Tr}\{(\not{p}_1 + m) \gamma_\nu (\not{p}'_1 - m) \gamma_\mu\} \\ &= \frac{4}{s^2} [p_2^\mu p'_{2\nu} + p_2^\nu p'_{2\mu} - (p_2 \cdot p'_2 + M^2) g^{\mu\nu}] [p_{1\mu} p'_{1\nu} + p_{1\nu} p'_{1\mu} - (p_1 \cdot p'_1 + m^2) g_{\mu\nu}] \\ &= \frac{8}{s^2} [(p_1 \cdot p_2)(p'_1 \cdot p'_2) + (p'_1 \cdot p_2)(p_1 \cdot p'_2) + M^2(p_1 \cdot p'_1) + m^2(p_2 \cdot p'_2) + 2m^2 M^2] \quad (0.3) \end{aligned}$$

In the c.m. frame $E_1 = E_2 = \frac{1}{2}\sqrt{s}$ and

$$\begin{aligned} p_1 \cdot p_2 &= p'_1 \cdot p'_2 = E_1 E_2 - |\vec{p}_1| |\vec{p}_2| \cos \theta, & p_1 \cdot p'_2 &= p'_1 \cdot p_2 = E_1 E_2 + |\vec{p}_1| |\vec{p}_2| \cos \theta \\ p_1 \cdot p'_1 &= \frac{s}{2} - m^2, & p_2 \cdot p'_2 &= \frac{s}{2} - M^2, & |\vec{p}_1| &= \frac{1}{2} \sqrt{s - 4m^2}, & |\vec{p}_2| &= \frac{1}{2} \sqrt{s - 4M^2} \end{aligned} \quad (0.4)$$

so we get

$$\begin{aligned} & \frac{1}{4e^4} \sum_{\lambda_1, \lambda'_1, \lambda_2, \lambda'_2} |T^{\lambda_1, \lambda'_1, \lambda_2, \lambda'_2}|^2 \\ &= \frac{4}{s^2} [4E_1^2 E_2^2 + 4\vec{p}_1^2 \vec{p}_2^2 \cos^2 \theta + s(M^2 + m^2)] \quad (0.5) \end{aligned}$$

and therefore

$$\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} &= \frac{1}{4} \sum_{\text{polarizations}} \frac{|T|^2}{64\pi^2 s} \frac{|\vec{p}_2|}{|\vec{p}_1|} \\
&= \frac{e^4}{16\pi^2 s^3} \frac{|\vec{p}_2|}{|\vec{p}_1|} [4E_1^2 E_2^2 + 4\vec{p}_1^2 \vec{p}_2^2 \cos^2 \theta + s(M^2 + m^2)]
\end{aligned} \tag{0.6}$$

The total cross section is given by

$$\begin{aligned}
\sigma_{\text{tot}} &= \int d\Omega \left(\frac{d\sigma}{d\Omega}\right) = \frac{e^4}{4\pi s^3} \frac{|\vec{p}_2|}{|\vec{p}_1|} [4E_1^2 E_2^2 + \frac{4}{3}\vec{p}_1^2 \vec{p}_2^2 + s(M^2 + m^2)] \\
&= \frac{e^4}{12\pi s} \left(1 + 2\frac{M^2 + m^2}{s} + \frac{4M^2 m^2}{s^2}\right) \sqrt{\frac{s - 4M^2}{s - 4m^2}}
\end{aligned} \tag{0.7}$$