HW assignment 3. Due March 3 at the lecture

For the theory of complex KG field

1. Find the differential and total cross section of elastic scattering of + and − particles (in the c.m. frame) in the leading order in perturbation theory.

2. Find the matrix element of M-matrix

$$\mathcal{M}(p_1+, p_1' - \quad \to p_2+, p_2'-)$$

in the λ^2 order in perturbation theory. (Do not calculate integrals over loop momentum, leave them as they stand).

Solution

First, we need to figure out the vertex in the set of Feynman rules . To this end, consider

$$\langle\Omega|\mathrm{T}\{\hat\phi(x_1)\hat\phi(x_2)\hat\phi^\dagger(y_1)\hat\phi^\dagger(y_2)\}|\Omega\rangle \;=\; \frac{\langle 0|\mathrm{T}\big\{e^{-i\int d^4z\ \frac{\lambda}{4}[\hat\phi_I^\dagger(z)\hat\phi_I(z)]^2}\,\hat\phi_I(x_1)\hat\phi_I(x_2)\hat\phi_I(y_1)\hat\phi_I(y_2)\big\}|0\rangle}{\langle 0|\mathrm{T}\big\{e^{-i\int d^4z\ \frac{\lambda}{4}[\hat\phi_I^\dagger(z)\hat\phi_I(z)]^2}\big\}||0\rangle}$$

In the leading order in perturbation theory in gives

$$\begin{aligned}
&-i\int d^4z\ \frac{\lambda}{4}\langle 0|\mathrm{T}\big\{\hat\phi_I^\dagger(z)\hat\phi_I^\dagger(z)\hat\phi_I^\dagger(z)\hat\phi_I^\dagger(z)\hat\phi_I(x_1)\hat\phi_I(x_2)\hat\phi_I(y_1)\hat\phi_I(y_2)\big\}|0\rangle\\
&= \ -i\lambda\int d^4z\,\overgroup{\hat\phi_I(x_1)\hat\phi_I^\dagger(z)}\,\overgroup{\hat\phi_I(x_2)\hat\phi_I^\dagger(z)}\,\overgroup{\hat\phi_I(z)\hat\phi_I^\dagger(y_1)}\,\overgroup{\hat\phi_I(z)\hat\phi_I^\dagger(y_2)}\\
&= \ -i\lambda\int d^4z D_F(x-z)D_F(x_2-z)D_F(z-y_1)D_F(z-y_2) \qquad (1)
\end{aligned}$$

From this equation we see tat the vertex for Feynman rules in the coordinate space is $-i\lambda(\times\int d^4z)$, same as for the real KG field. By analogy with real KG field we see that the vertex in a set of Feynman rules for reduced Green functions is $-\lambda$ and therefore the first-order amputated reduced Green function for $2\to 2$ particle scattering is $-\lambda$.

The lowest-order cross section is

$$\frac{d\sigma}{d\Omega} \;=\; \frac{|M|^2}{64\pi^2 s} \;=\; \frac{\lambda^2}{64\pi^2 s} \qquad (2)$$

and the total cross section is

$$\sigma_{\rm tot} \;=\; \int d\Omega\frac{|M|^2}{64\pi^2 s} \;=\; \frac{\lambda^2}{16\pi s} \qquad (3)$$

In the next order in perturbation theory there are three diagrams

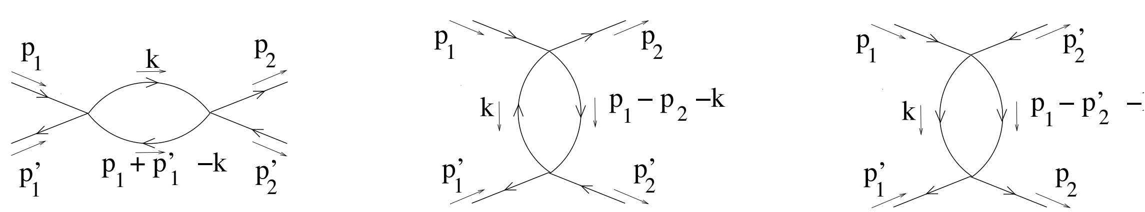

FIG. 1: Diagramms for $\mathcal{M}$ matrix in λ^2 order

$$\begin{aligned}
\mathcal{G}^{\rm amp}(p_1,p_1'\to p_2,p_2') \;=\; &-\lambda\int \bar{d}^4p\,\Big[\frac{1}{(m^2-p^2-i\epsilon)[m^2-(p_1+p_1'-k)^2-i\epsilon]}+\frac{1}{(m^2-p^2-i\epsilon)[m^2-(p_1-p_2-k)^2-i\epsilon]}\\
&+\frac{1}{(m^2-p^2-i\epsilon)[m^2-(p_1-p_2'-k)^2-i\epsilon]}\Big] \qquad (4)
\end{aligned}$$