HW assignment 6
Due Tue Apr. 19 at the lecture

Consirer Yukawa theory with the Lagrangian (17.1) but with massless π-meson

$$\mathcal{L} \;=\; \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{M^2}{2}\phi^2 + \bar\psi\big(i\gamma^\mu\partial_\mu - m\big)\psi(x) - g\phi\bar\psi\psi\Big|_{M^2=0} \;=\; \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \bar\psi\big(i\gamma^\mu\partial_\mu - m\big)\psi(x) - g\phi\bar\psi\psi$$

The bare propagator will be massless, $\mathcal{G}_{(0)}(p) \;=\; \frac{1}{-p^2-i\epsilon}$.

Working in terms of unrenormalized fields and bare g_0 and m_0, calculate the δM correction in the leading order in g^2 and show that the exact pion propagator does not have a pole at $p^2 = 0$. What would be the expression for pion mass in the first non-trivial order in perturbation theory (in terms of g_0, m_0 and the UV cutoff)?. Will the location of this pole be tachionic ($M^2 < 0$) or normal($M^2 > 0$)?