High-energy scattering in Quantum Chromodynamics

Ian Balitsky

ODU& JLab

ODU, 24 Jan 06

Plan

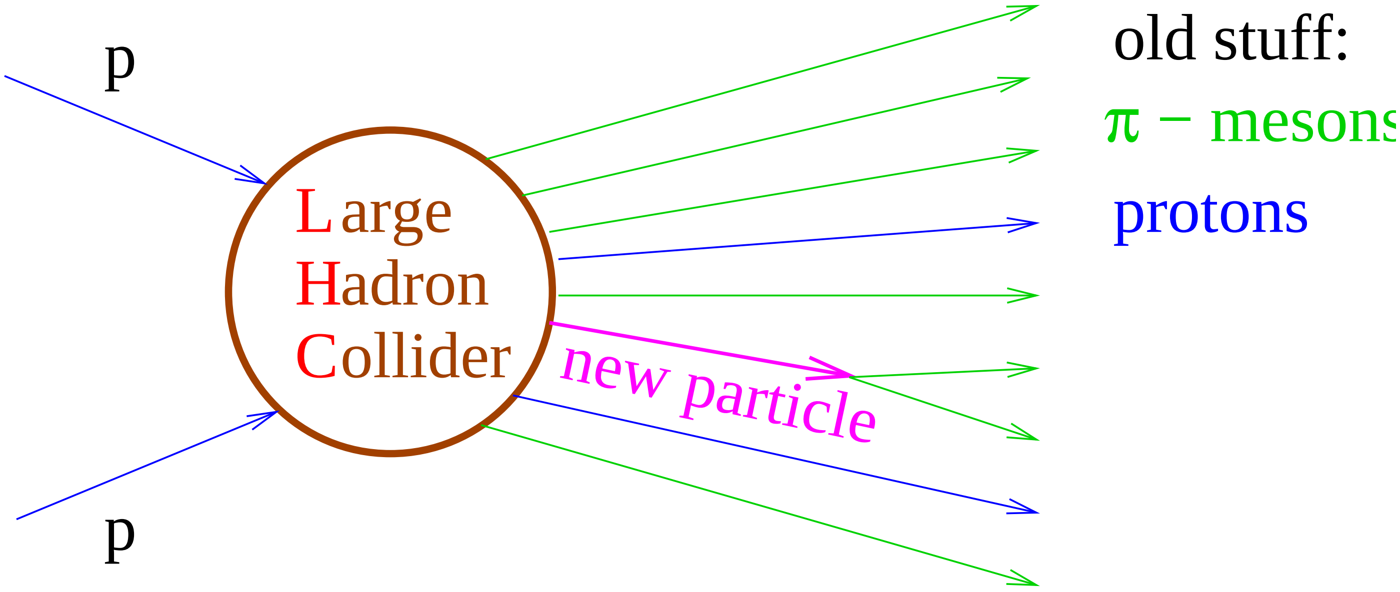
- Why high energy?
- QCD - a theory of strong interactions
- Feynman diagrams in QED and QCD
- Asymptotic freedom and hadron-hadron scattering in QCD
- Deep inelastic scatteing - *the* experiment for QCD
- DIS at small x
- High-energy scattering in pQCD and the BFKL pomeron
- Wilson-line approach to high-energy scattering in quantum mechanics and QCD
- Non-linear evolution of color dipoles
- Parton saturation at high energy
- Saturation and heavy-ion collisions
- Conclusions and outlook

High-energy scattering as a probe of new physics

Heisenberg uncertainty principle: $\Delta x = \frac{\hbar}{p} = \frac{\hbar c}{E}$

LHC(coming 2007): E=7 TeV $\longrightarrow$ distances $\sim 10^{-18}$ cm

(Planck scale is 10^{-33} cm - a long way to go!)



To separate a “new physics signal” from the “old” background one needs to understand the behavior of QCD cross sections at large energies

Strong interactions at asymptotic energies: Froissart bound

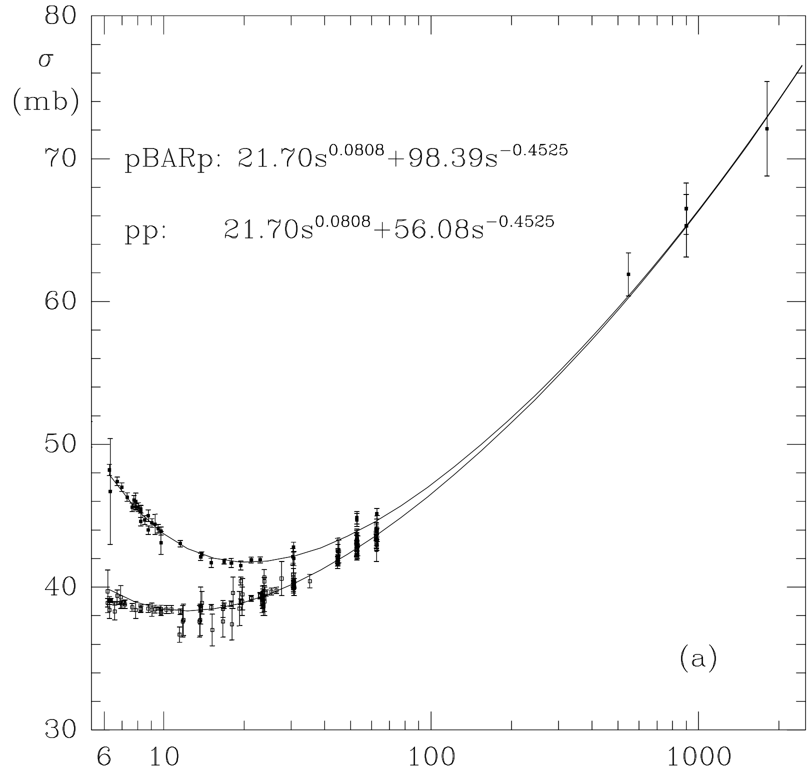
Regge limit: $E \gg$ everything else

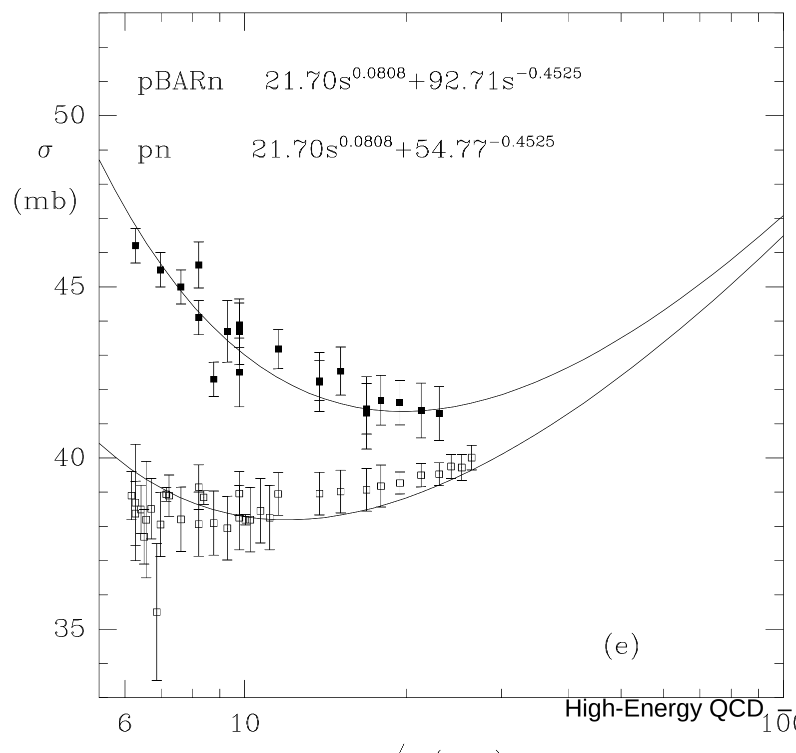
$$\left.\begin{array}{l}\text{Causality}\\ \text{Unitarity}\end{array}\right\} \quad \Rightarrow \quad \sigma_{\rm tot} \overset{E\to\infty}{\leq} \ln^2 E$$

Froissart, 1962

Long-standing problem - not explained in any quantum field theory (or string theory) in almost 50 years!

Experiment: $\sigma_{\rm tot} \sim s^{0.08}$ ($s \equiv 4E^2_{\rm c.m.}$). Numerically close to $\ln^2 E$ - see below





Quantum ChromoDynamics: a theory of strong interactions

$$L = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_{\text{flavors}} \bar{q}^k (i\gamma_\mu D^\mu + m_q)^{kl} q^l$$

Enjoy the beauty!

Quantum ChromoDynamics: a theory of strong interactions

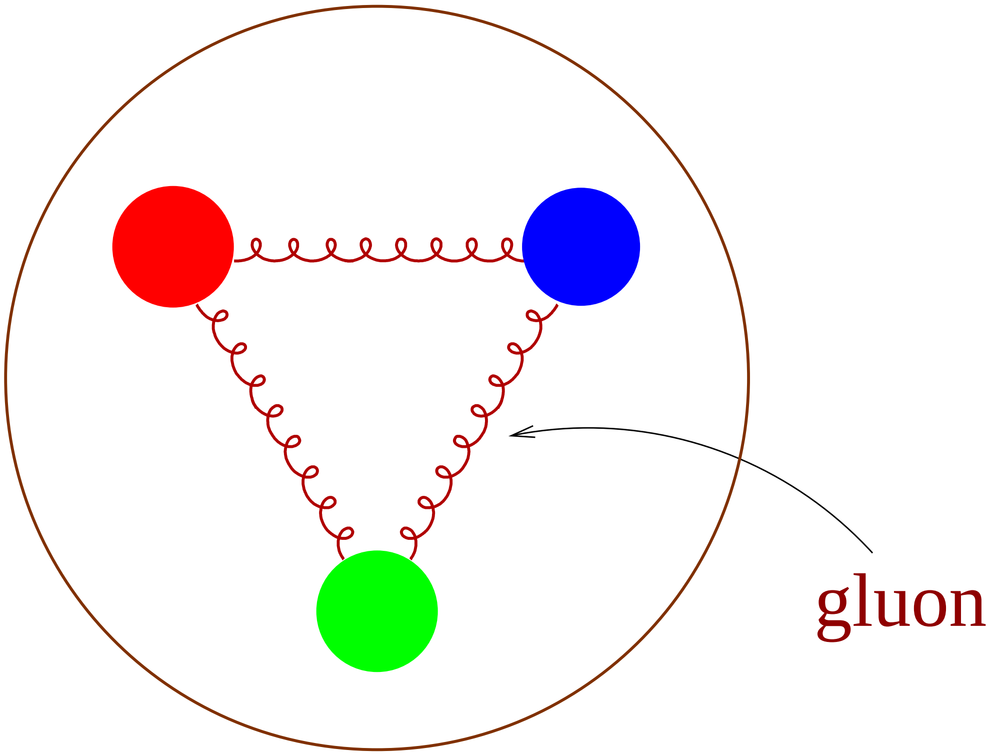
$$L = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_{\text{flavors}} \bar{q}^k (i\gamma_\mu D^\mu + m_q)^{kl} q^l$$

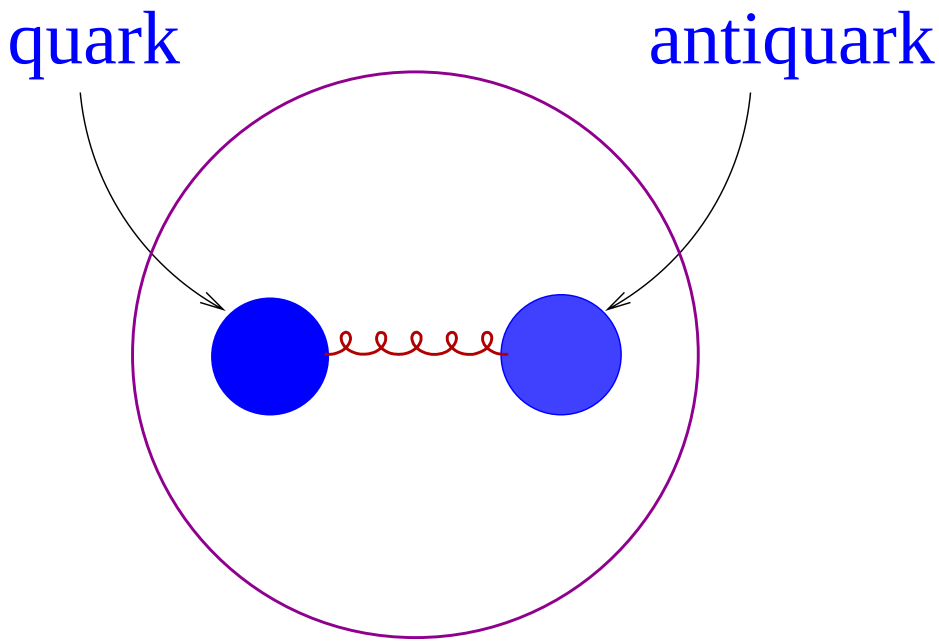
Enjoy the beauty!

QCD describes:

- **Quarks and anti-quarks (spin $\frac{1}{2}$ fermions)**
  - 3 colors (red, blue, and green)
  - 6 flavors (u,d,s,c,b,t)
- **Gluons (spin 1 bosons)**
  - 8 colors - pairwise combinations of quark clolors (e.g. red and anti-blue)

Hadrons: colorless states of quarks bound by gluons





Baryons
(proton, neutron, ...)

Mesons
(pion, kaon, ρ , ...)

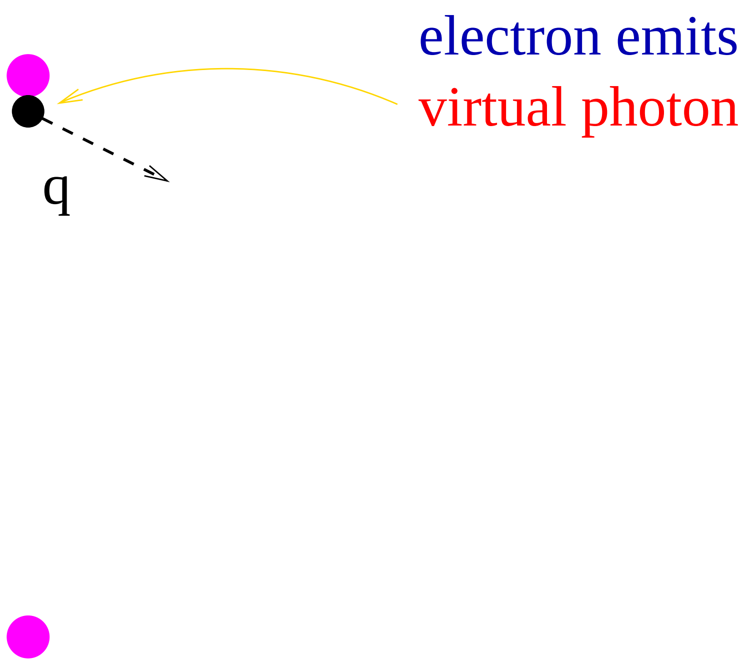
Electon scattering in Quantum ElectroDynamics

Electon scattering in Quantum ElectroDynamics

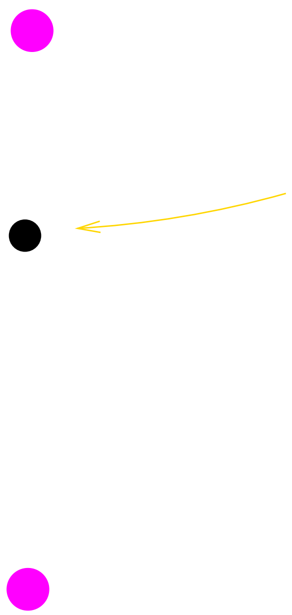
Electon scattering in Quantum ElectroDynamics

Virtual particle: particle with $E \neq \sqrt{m^2c^4 + p^2c^2}$.

Lives for a short time $\Delta t \sim \frac{\hbar}{E - \sqrt{m^2c^4+p^2c^2}}$



Electon scattering in Quantum ElectroDynamics



virtual photon with
virtuality $q_\mu q^\mu = q^2 = -Q^2$
lives for a short time $t \sim 1/Qc$

Electon scattering in Quantum ElectroDynamics

Electon scattering in Quantum ElectroDynamics

virtual photon is absorbred
by the second electron

Electon scattering in Quantum ElectroDynamics

Electon scattering in Quantum ElectroDynamics

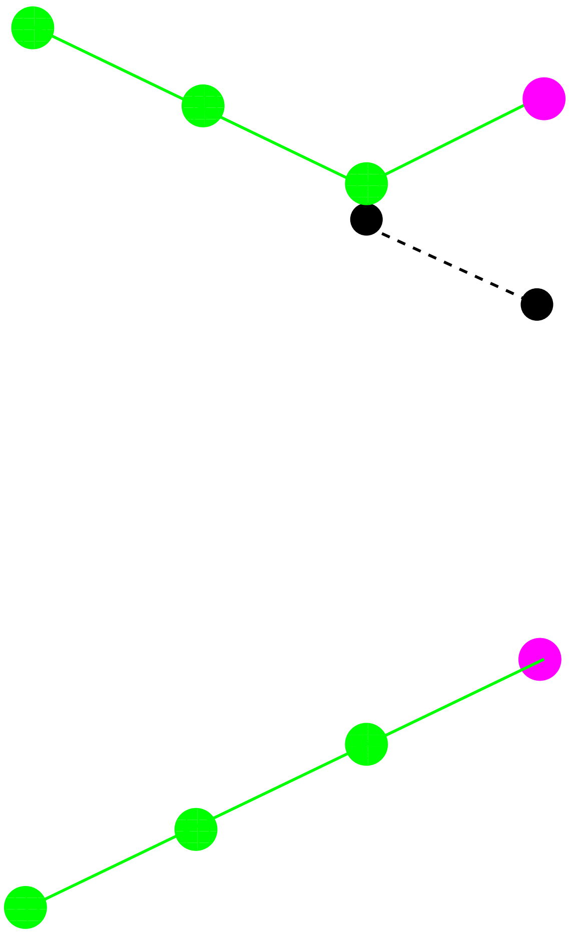
-

-

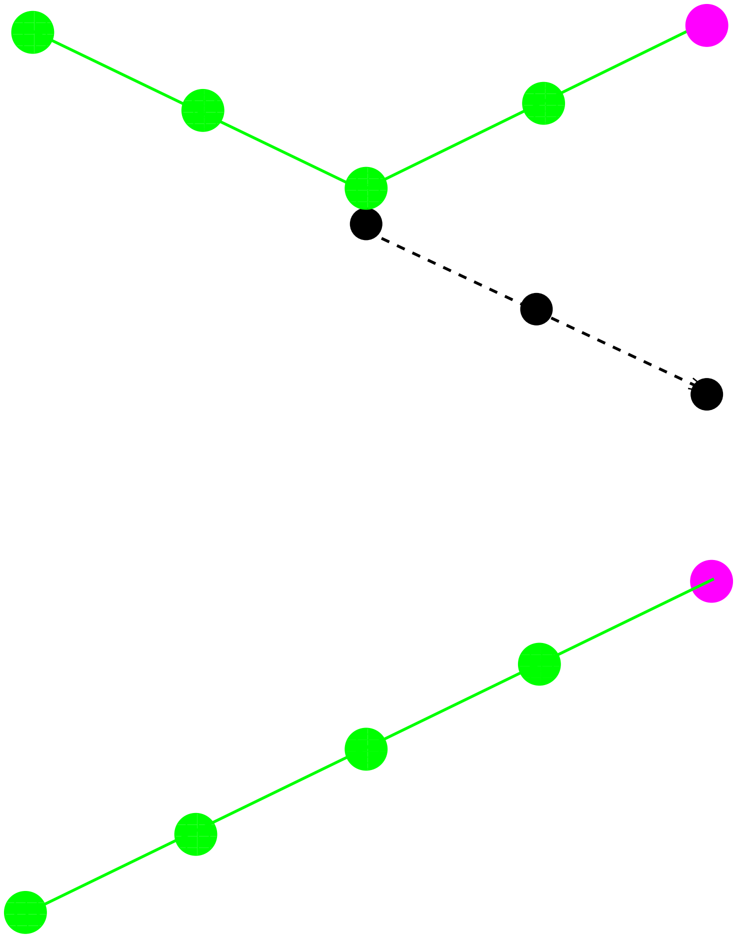
Feynman diagrams


6

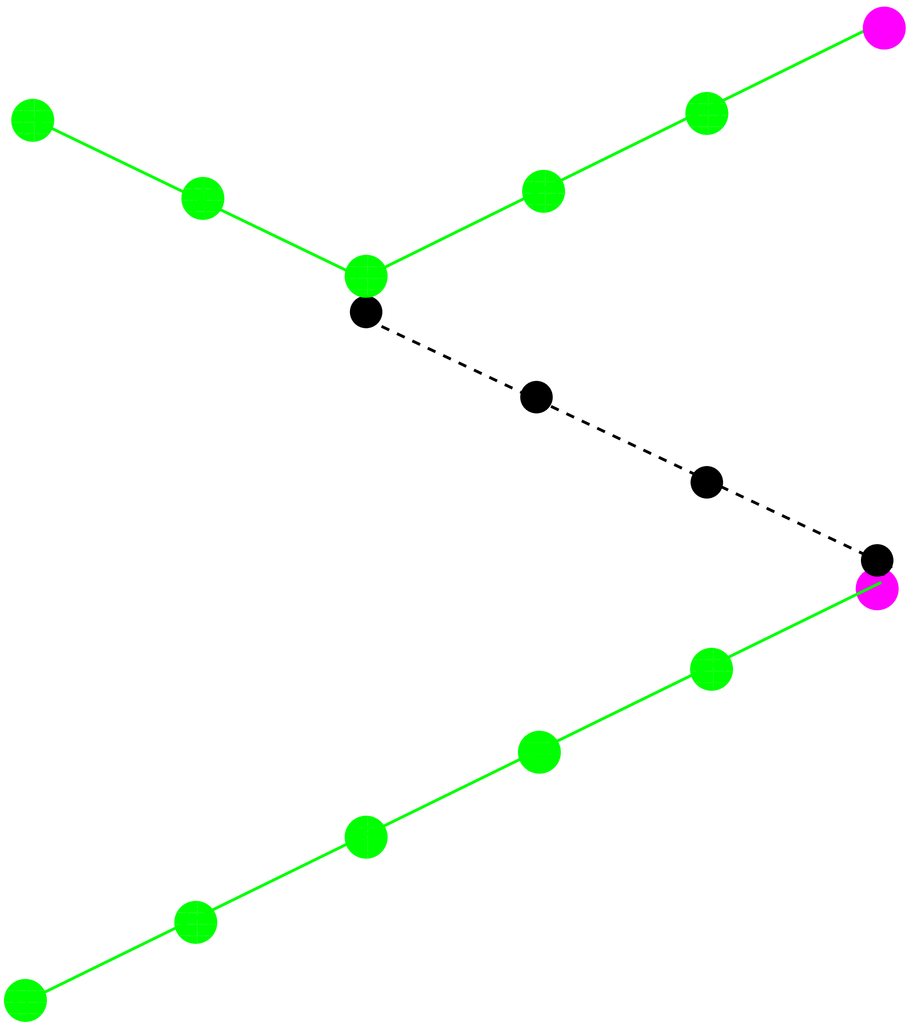
Feynman diagrams



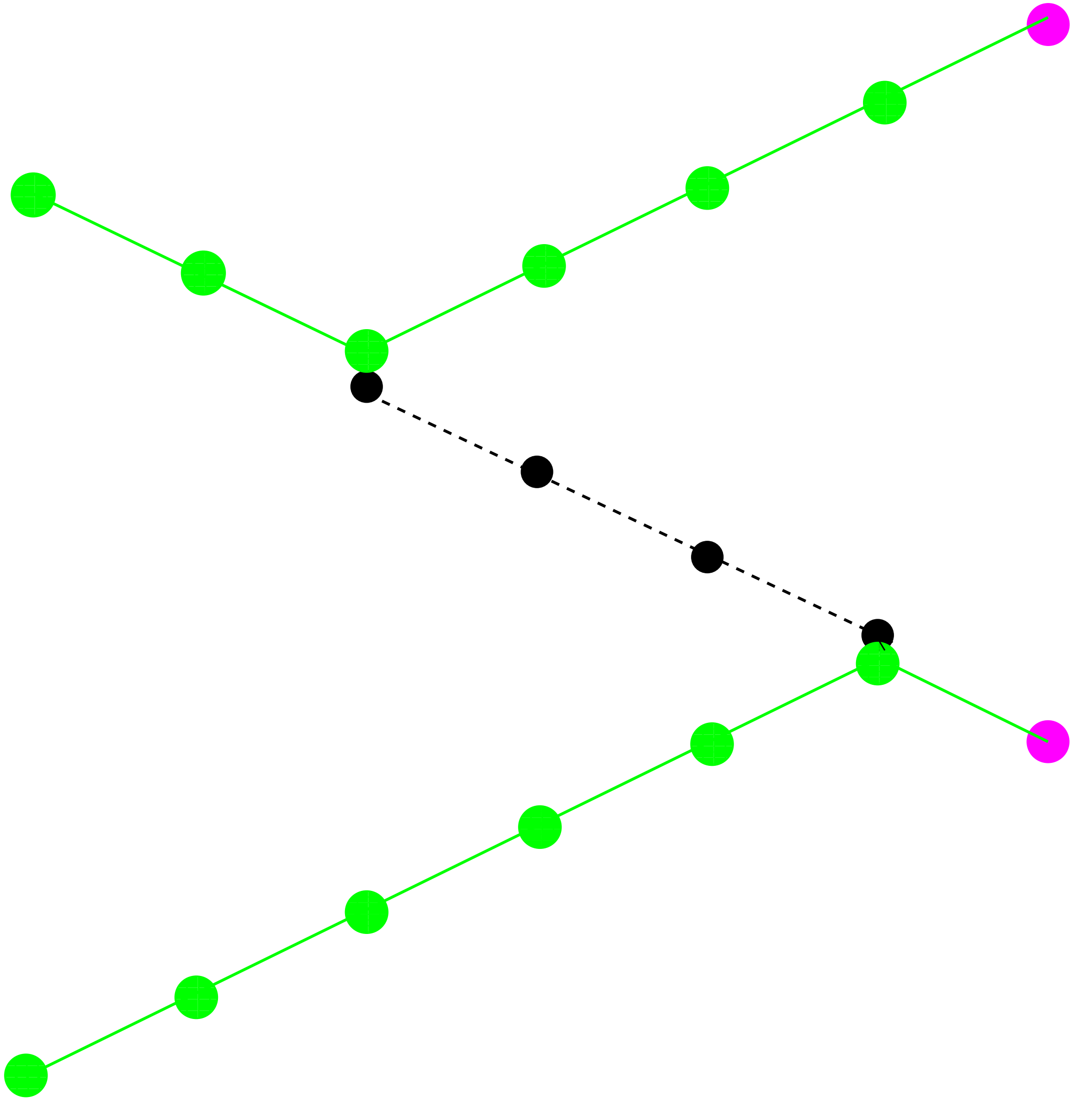
Feynman diagrams



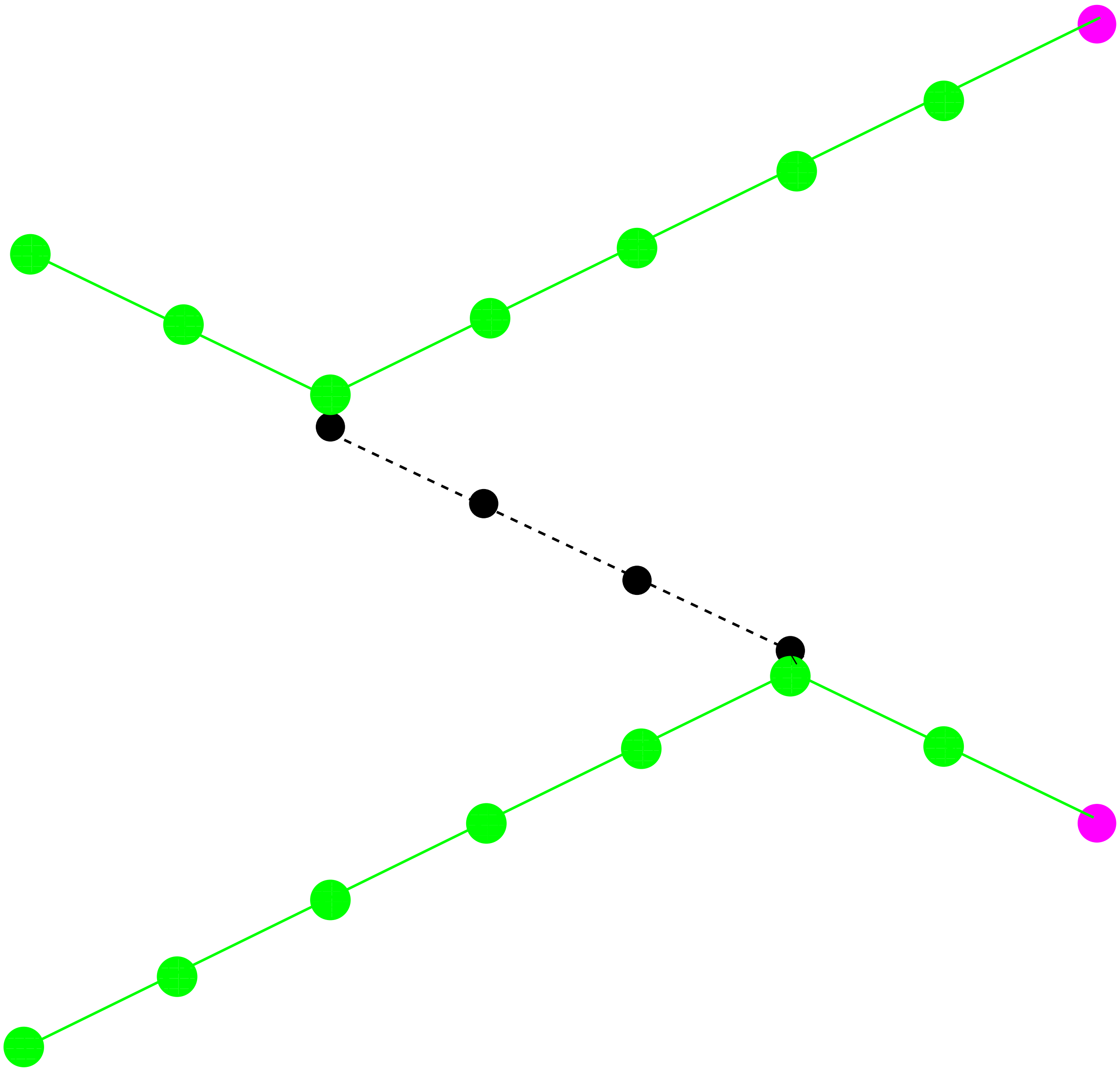
Feynman diagrams



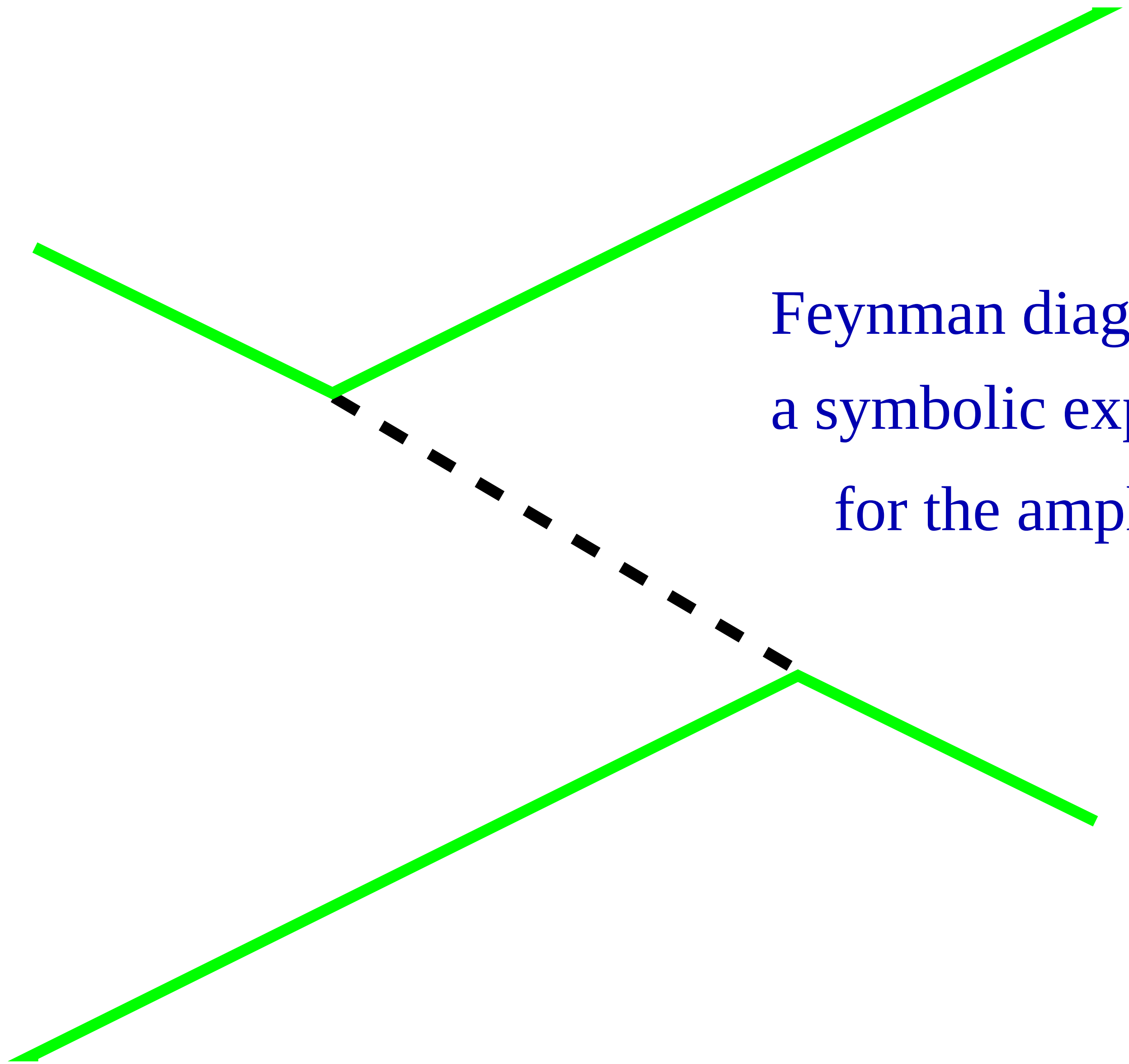
Feynman diagrams



Feynman diagrams



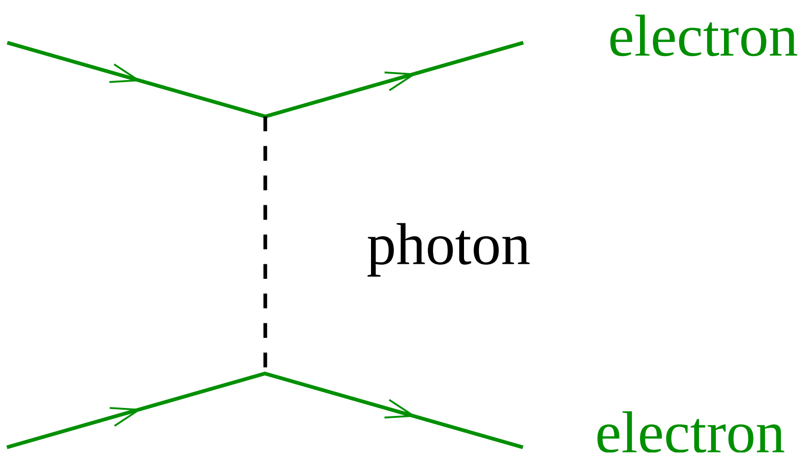
Feynman diagrams



Feynman diagram:

a symbolic expression

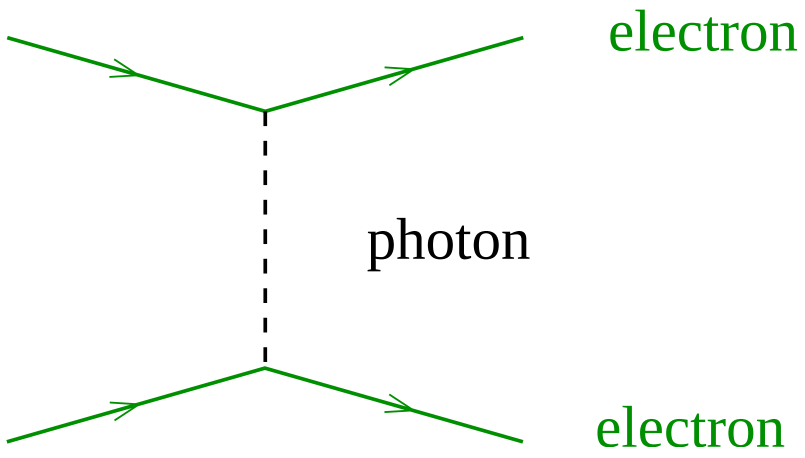
for the amplitude



In QED interactions of electrons are mediated by photons.

Photons do not interact with each other.

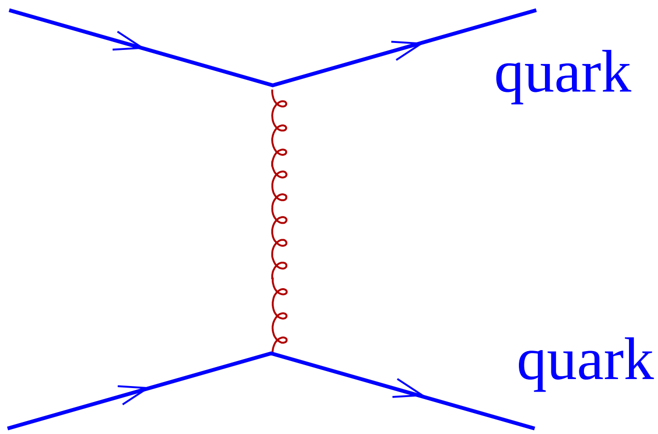
QCD *versus* QED

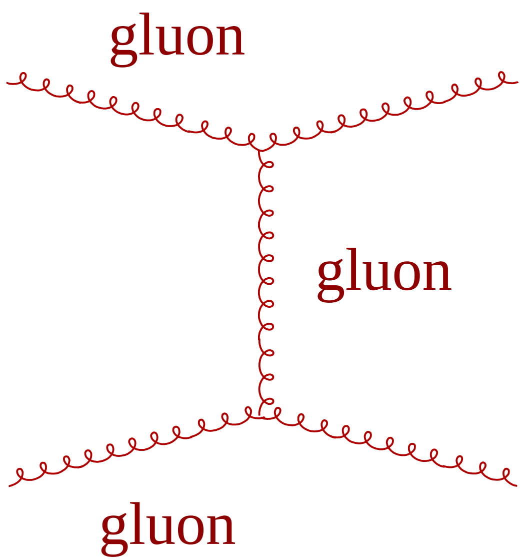


In QED interactions of electrons are mediated by photons.

Photons do not interact with each other.

In QCD

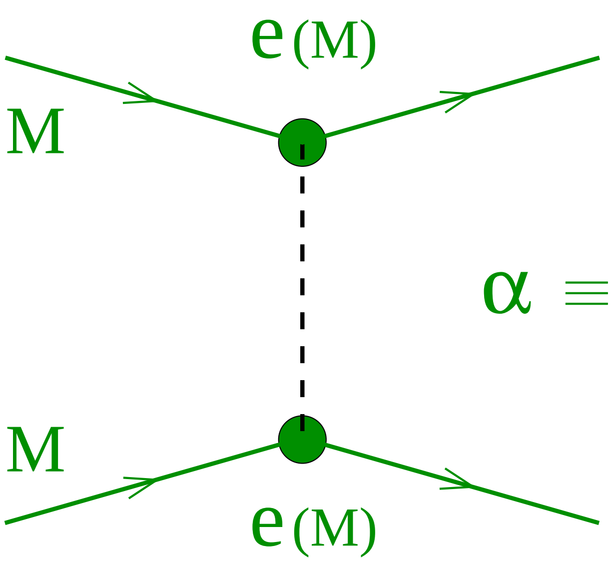




Quarks interact by exchanging gluons, but gluons can also interact with each other!

$\Rightarrow$ QCD is a non-linear theory

QED:



$$\alpha \equiv \frac{e^2}{4\pi\hbar c} \simeq \frac{1}{137}$$

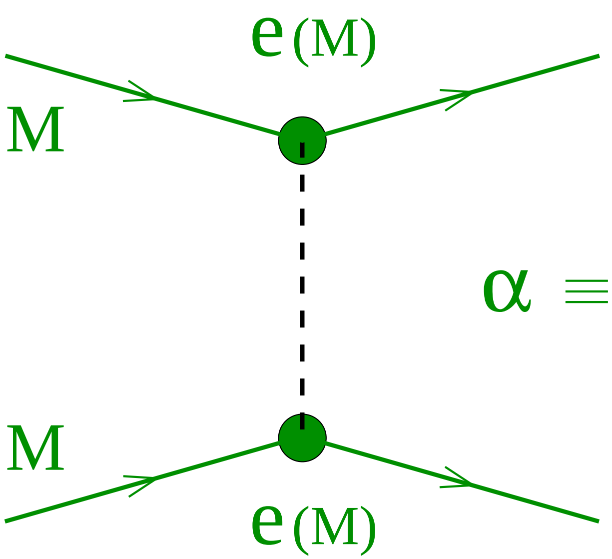
The strength of the interaction depend on the mass scale

$$\alpha(M) = \frac{\alpha(m)}{1 - \frac{\alpha(m)}{3\pi} \ln \frac{M^2}{m^2}}$$

$\alpha(M)$ increases as $M \to \infty$

QCD *versus* QED

QED:



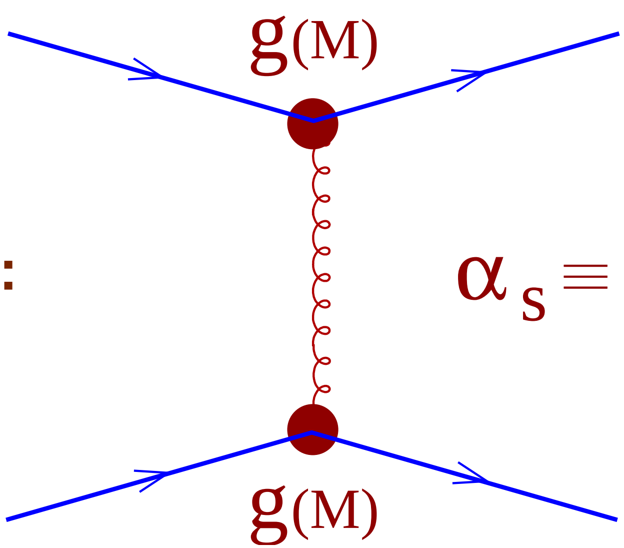
$$\alpha \equiv \frac{e^2}{4\pi\hbar c} \simeq \frac{1}{137}$$

The strength of the interaction depend on the mass scale

$$\alpha(M) = \frac{\alpha(m)}{1 - \frac{\alpha(m)}{3\pi} \ln \frac{M^2}{m^2}}$$

$\alpha(M)$ increases as $M \to \infty$

QCD:



$$\alpha_s \equiv \frac{g^2}{4\pi\hbar c}$$

$$\alpha_s(M) = \frac{\alpha_s(m)}{1 + \frac{b\alpha_s(m)}{4\pi} \ln \frac{M^2}{m^2}}$$

$$b = 11 - \tfrac{2}{3} n_f$$

$\Rightarrow \alpha(M) \to 0$ as $M \to \infty$

Asymptotic freedom

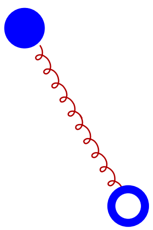
Gross, Politzer, Wilczek (1973)

Nobel Prize 2004

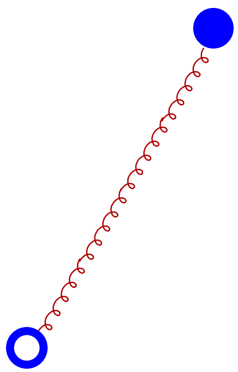
Asymptotic Freedom and Quark Confinement

Short Distances/Large Momenta $\Rightarrow$ Small Coupling $\Rightarrow$ Asymp. Freedom
Large Distances/Small Momenta $\Rightarrow$ Large Coupling $\Rightarrow$ Confinement

A toy model for a hadron - two quarks connected by a spring with a small k



at small distances force is weak − quarks are almost free



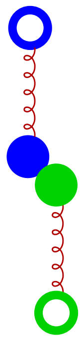
at large distances force increases − confinement

Meson-meson scattering at high energies

Meson-meson scattering at high energies

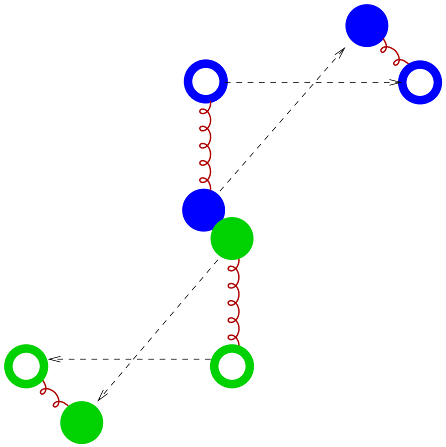
Meson-meson scattering at high energies

Meson-meson scattering at high energies

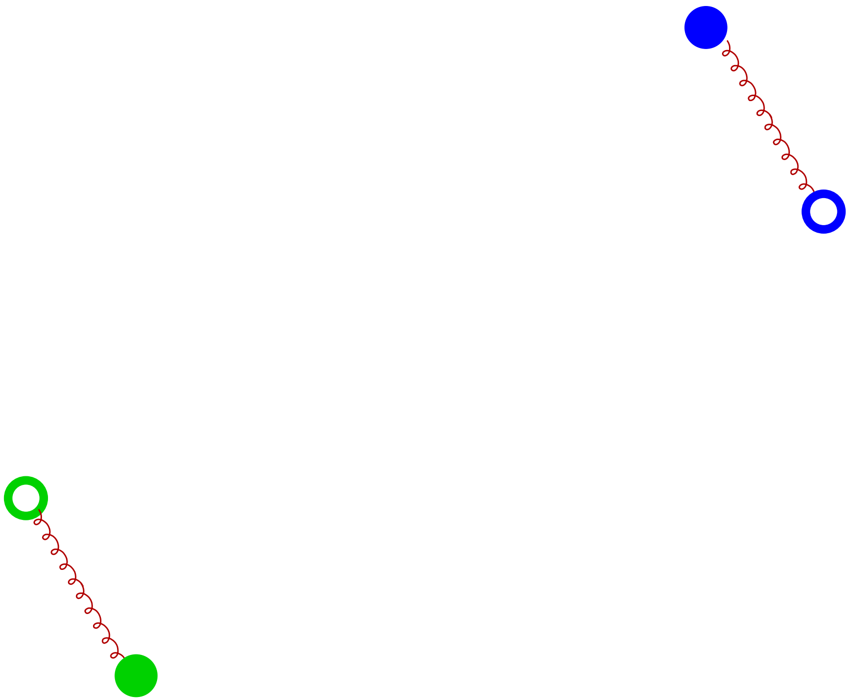


Meson-meson scattering at high energies

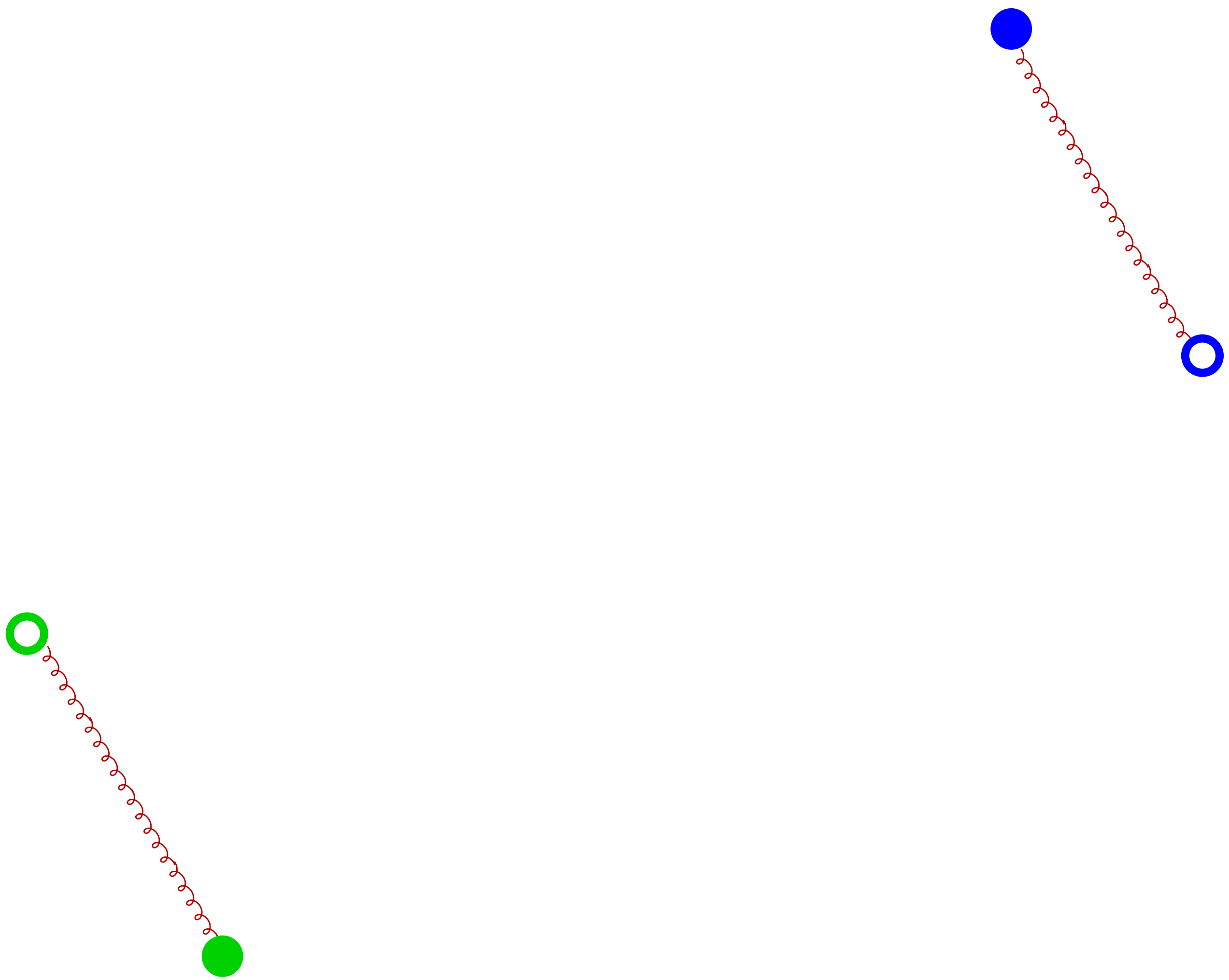
Hard quark−quark scattering



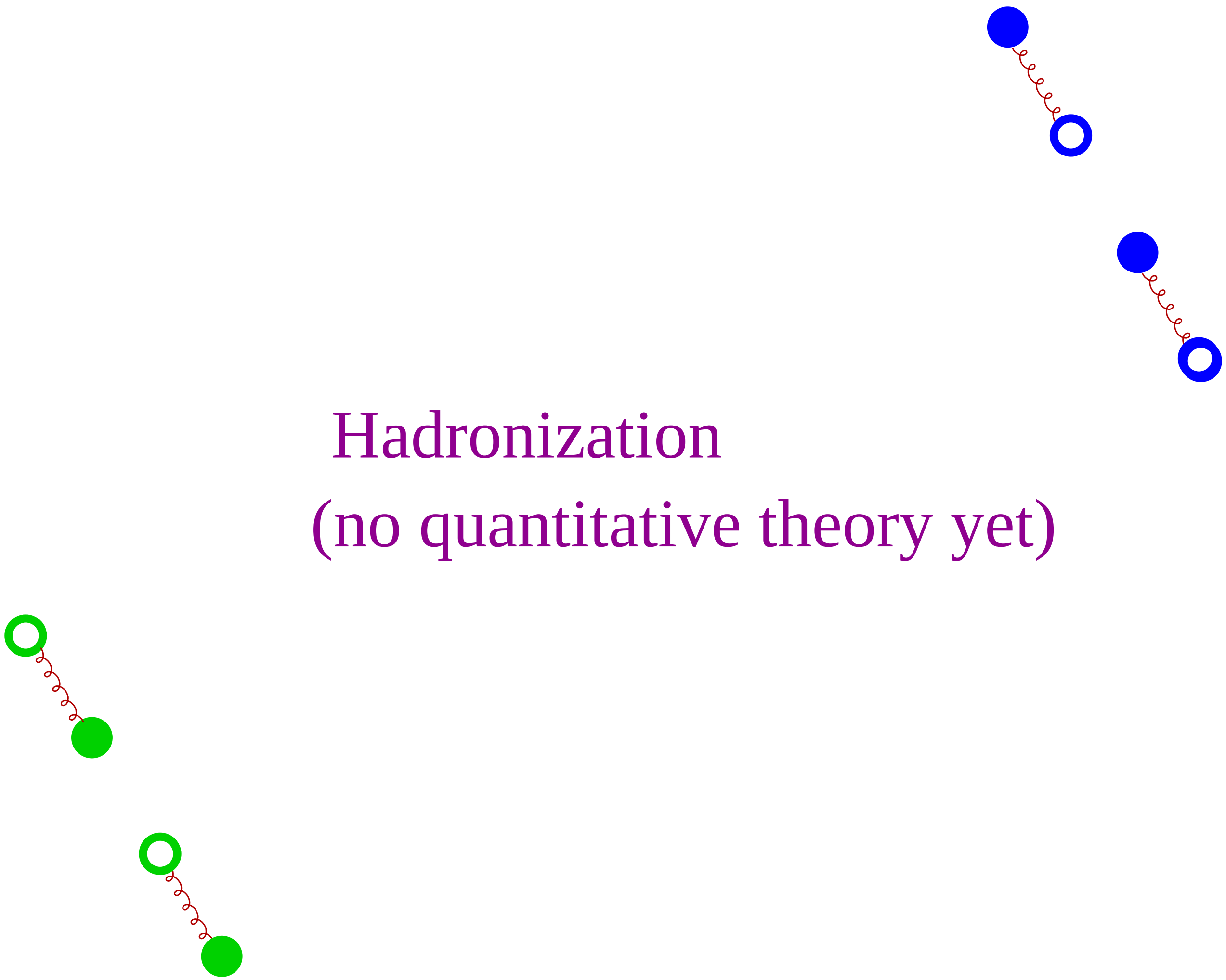
Meson-meson scattering at high energies



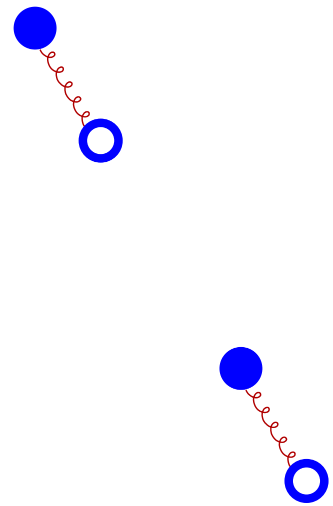
Meson-meson scattering at high energies



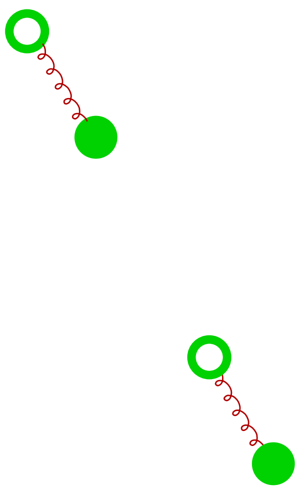
Meson-meson scattering at high energies



Meson-meson scattering at high energies



''Soft'' hadronization:
all momenta distributions
of final hadrons are
determined by the hard
quark−quark scattering



Deep inelastic scattering (DIS) - *the* experiment for QCD

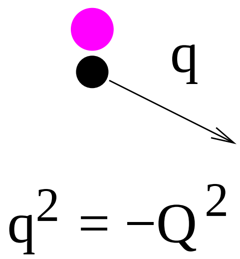
Regge limi in DIS: $E \gg Q \equiv \ x \ll 1$ $\qquad x = \frac{Q^2}{2p \cdot q}$ - Bjorken variable

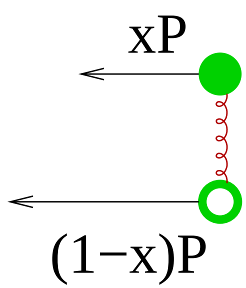
Deep inelastic scattering from a meson

Deep inelastic scattering from a meson

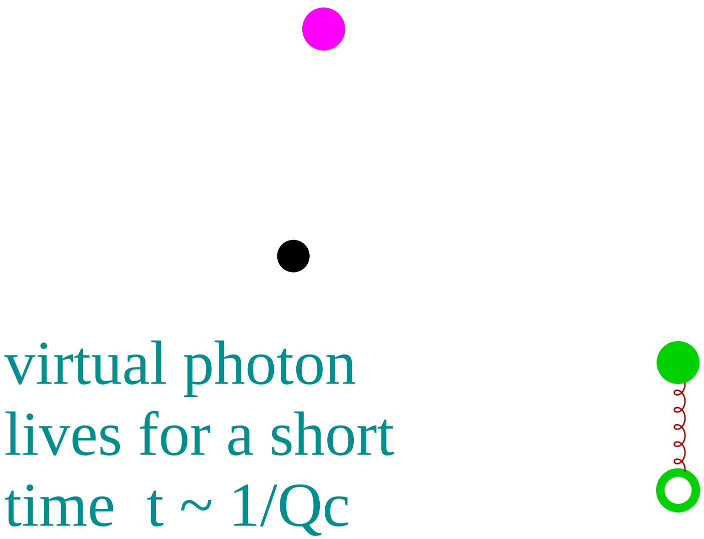
Deep inelastic scattering from a meson

electron emits virtual photon

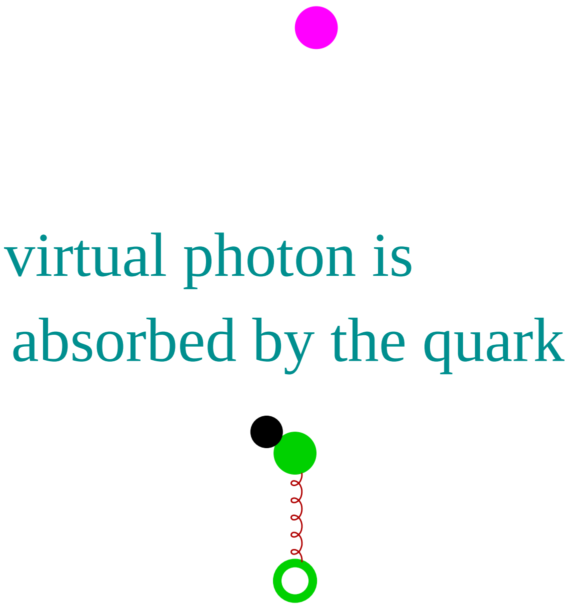




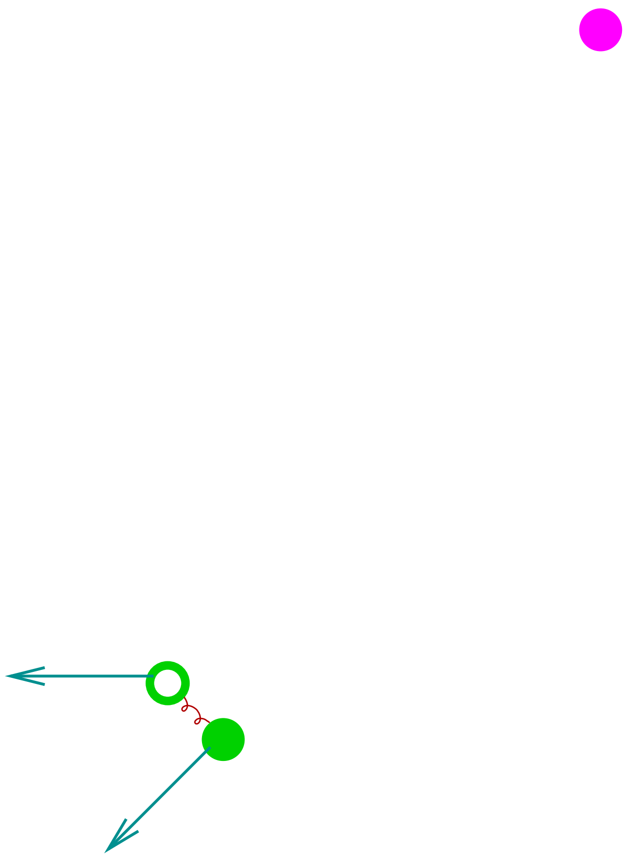
Deep inelastic scattering from a meson



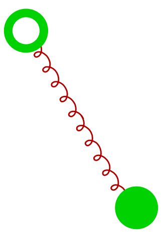
Deep inelastic scattering from a meson



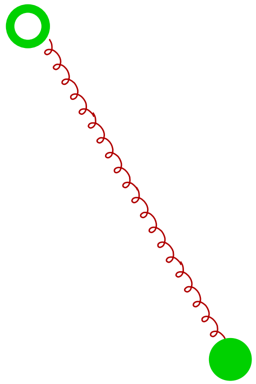
Deep inelastic scattering from a meson



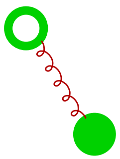
Deep inelastic scattering from a meson



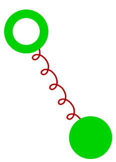
Deep inelastic scattering from a meson



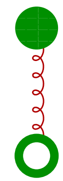
Deep inelastic scattering from a meson



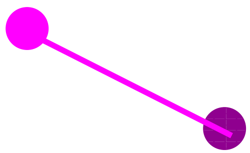
Hadronizaton

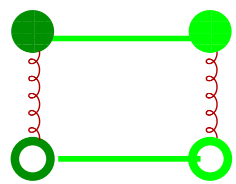


Feynman diagrams for DIS

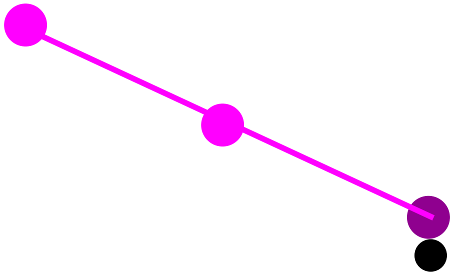


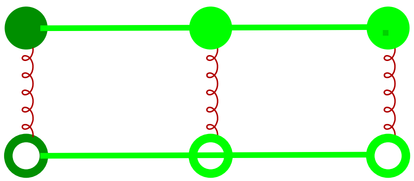
Feynman diagrams for DIS

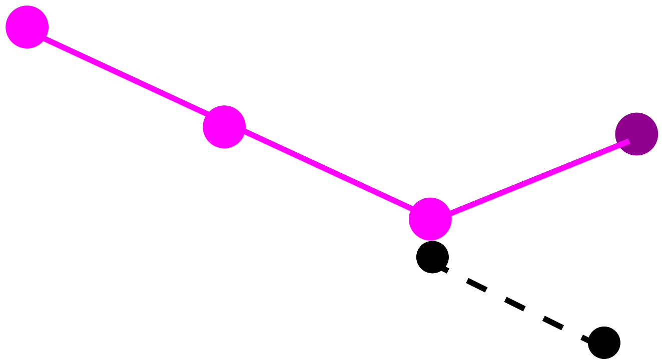


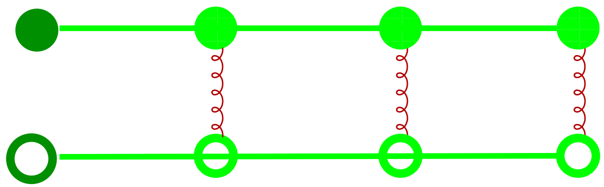


Feynman diagrams for DIS

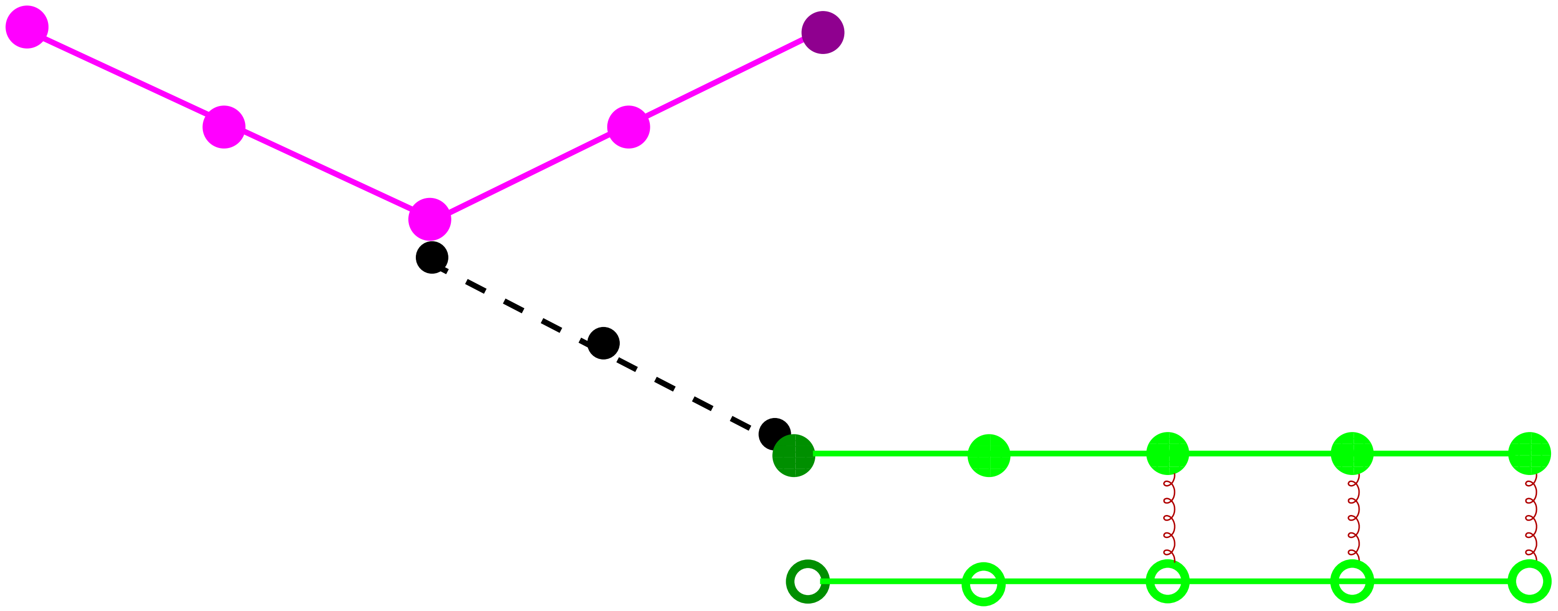


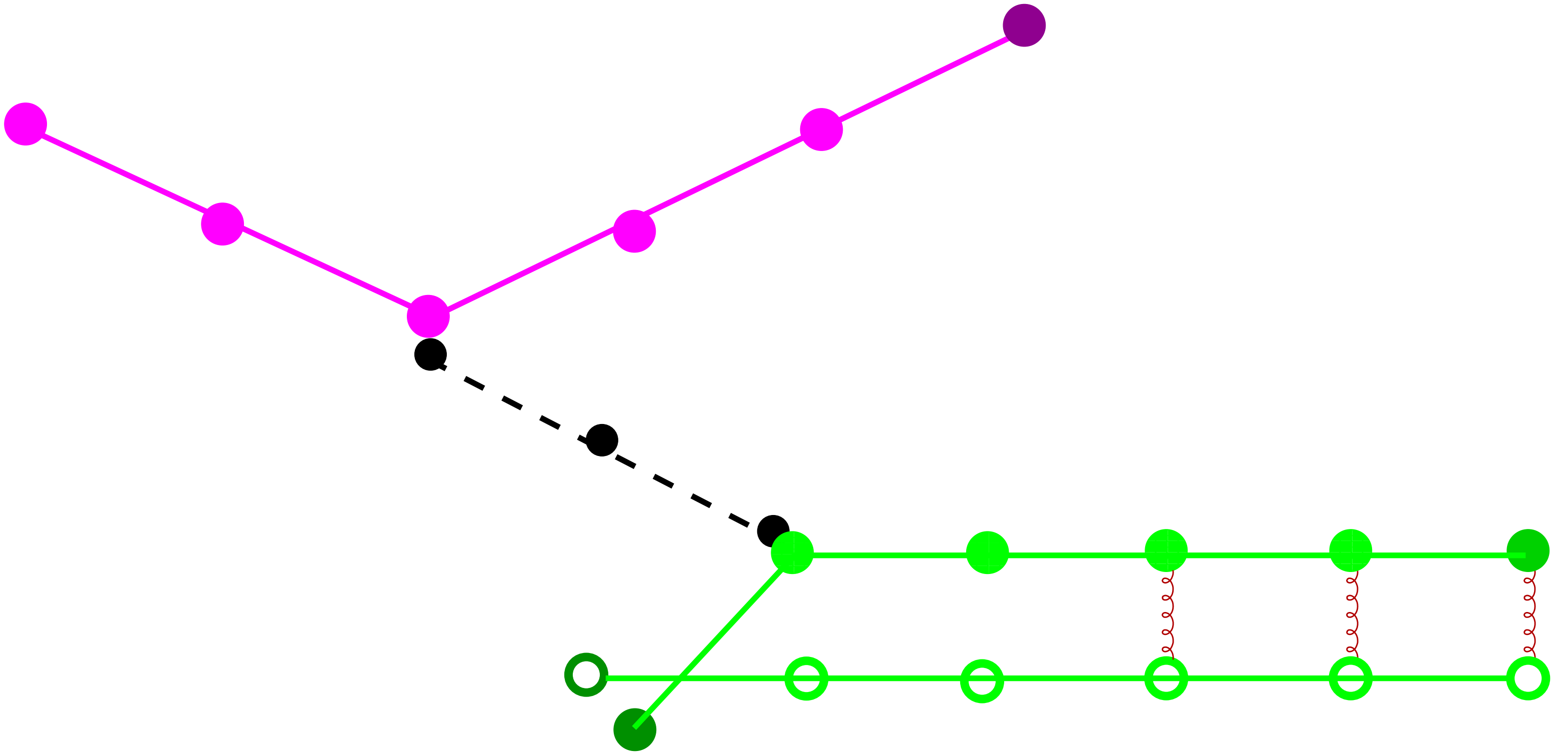




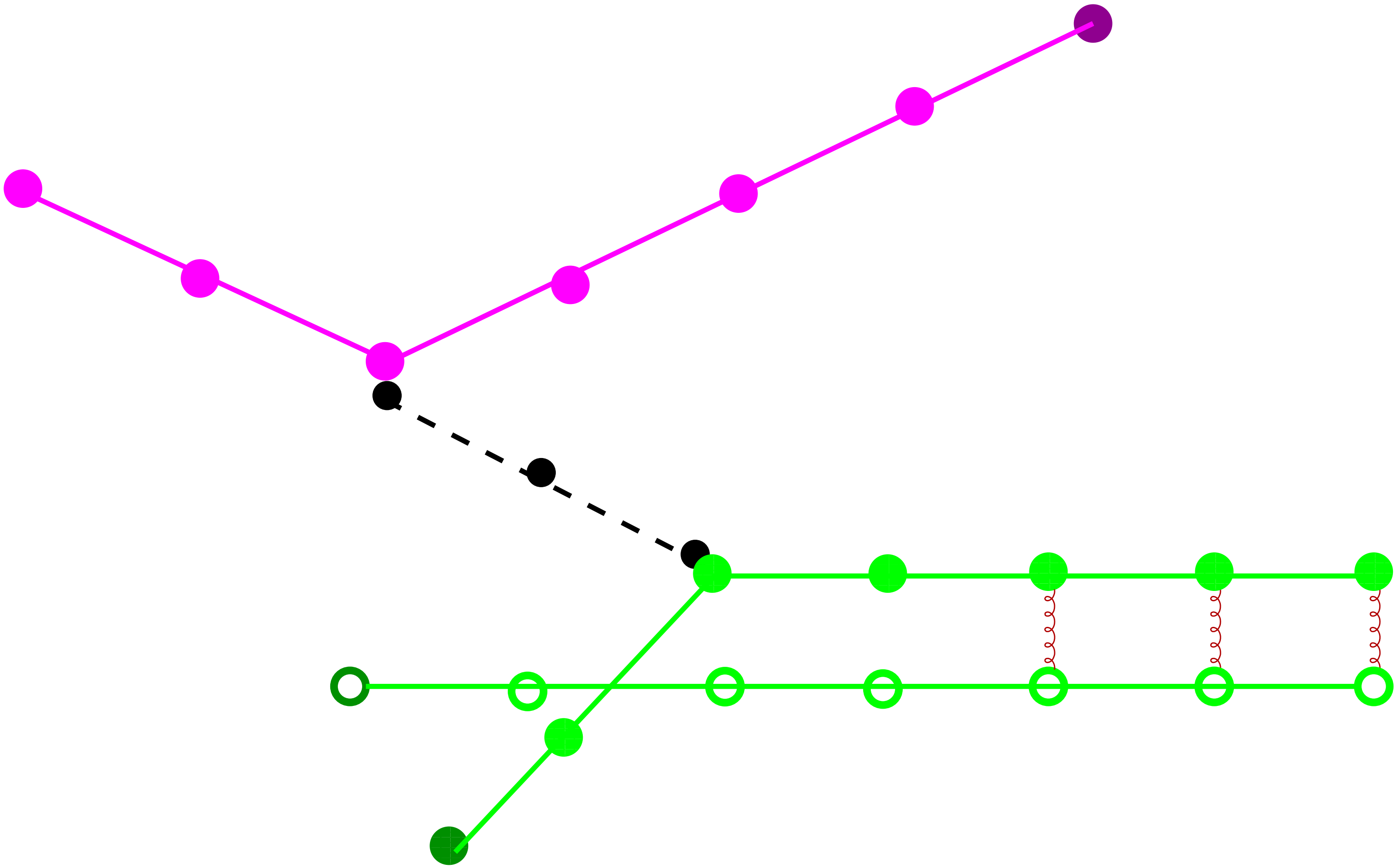


Feynman diagrams for DIS

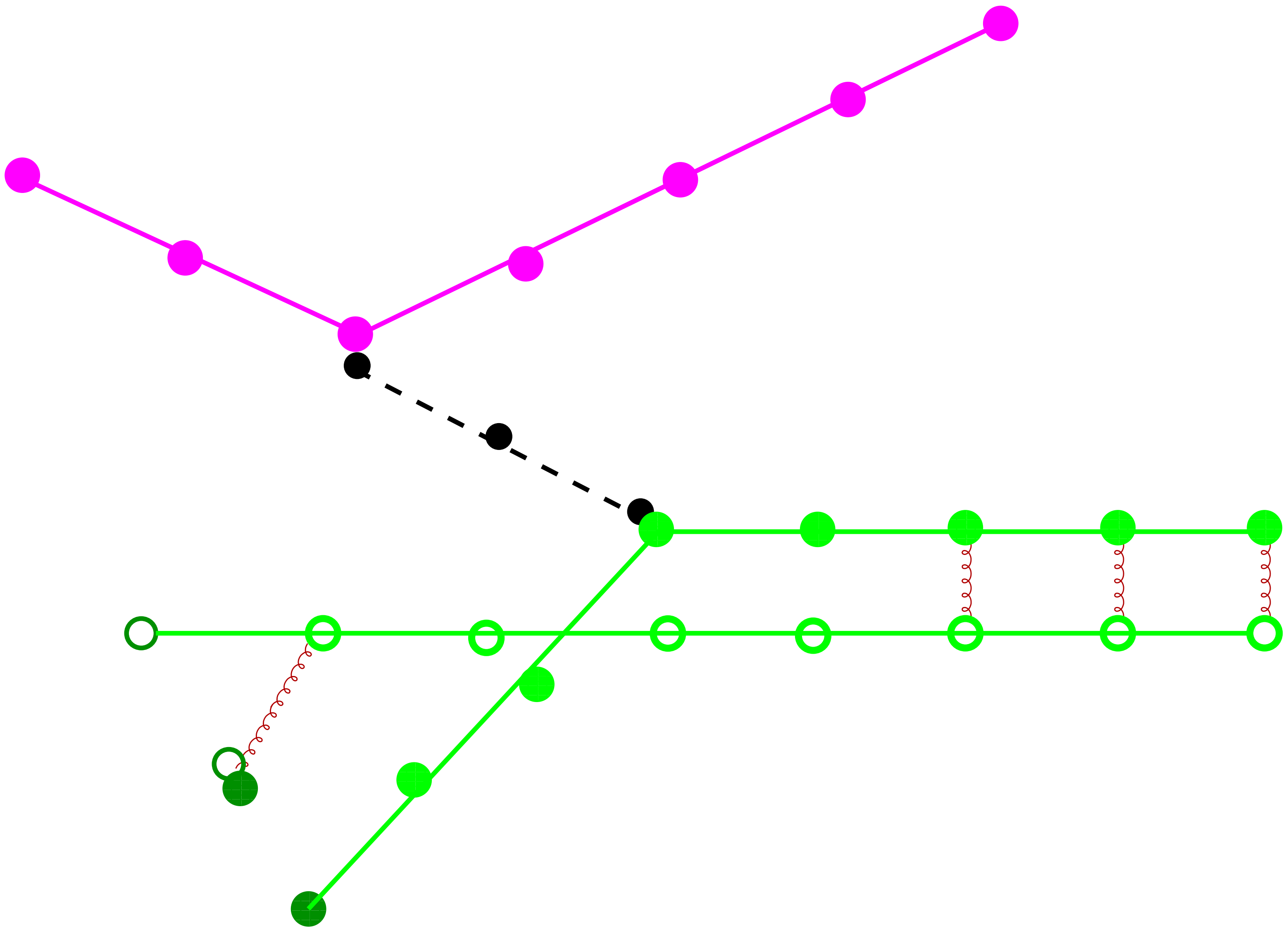


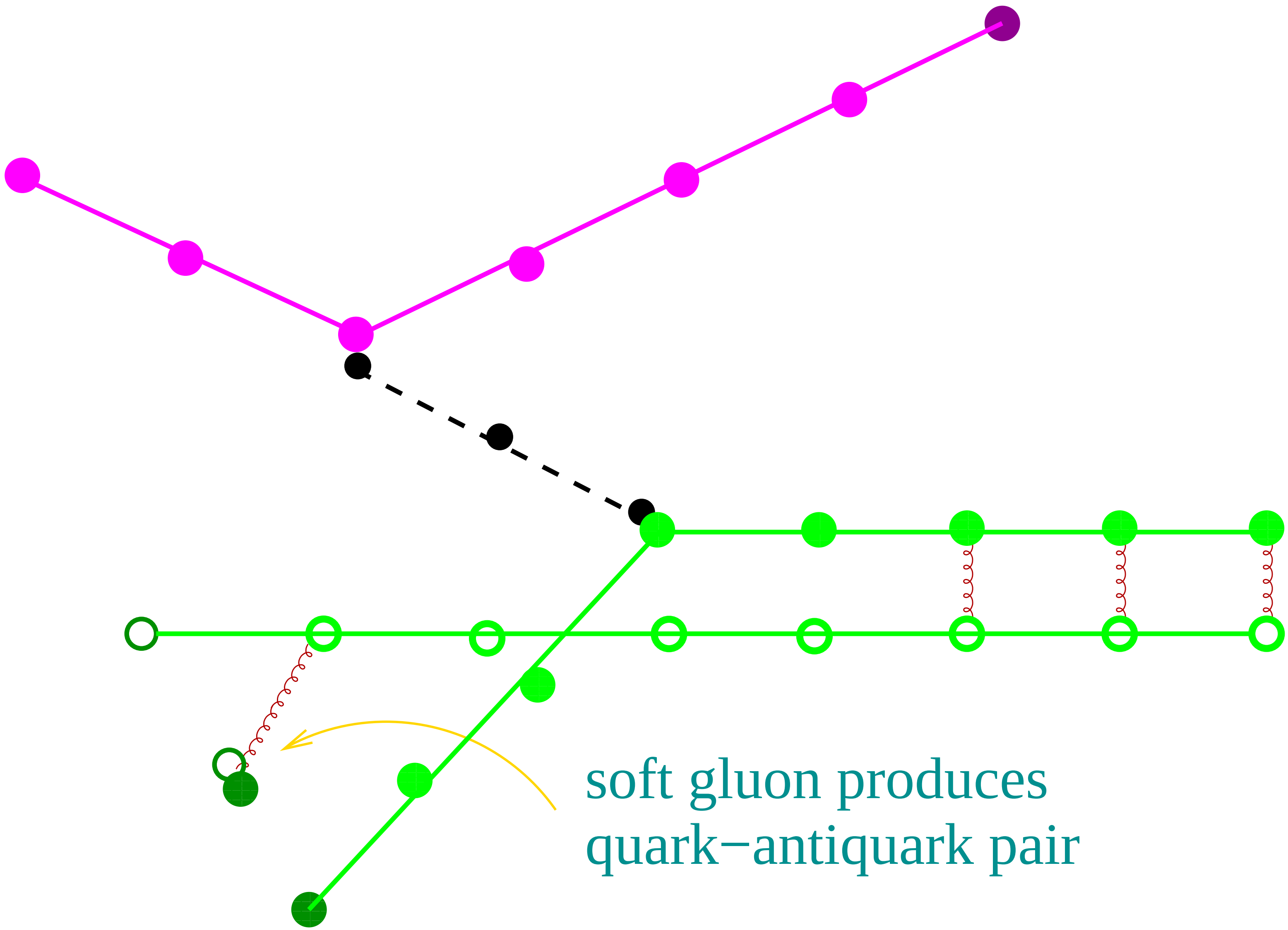


Feynman diagrams for DIS

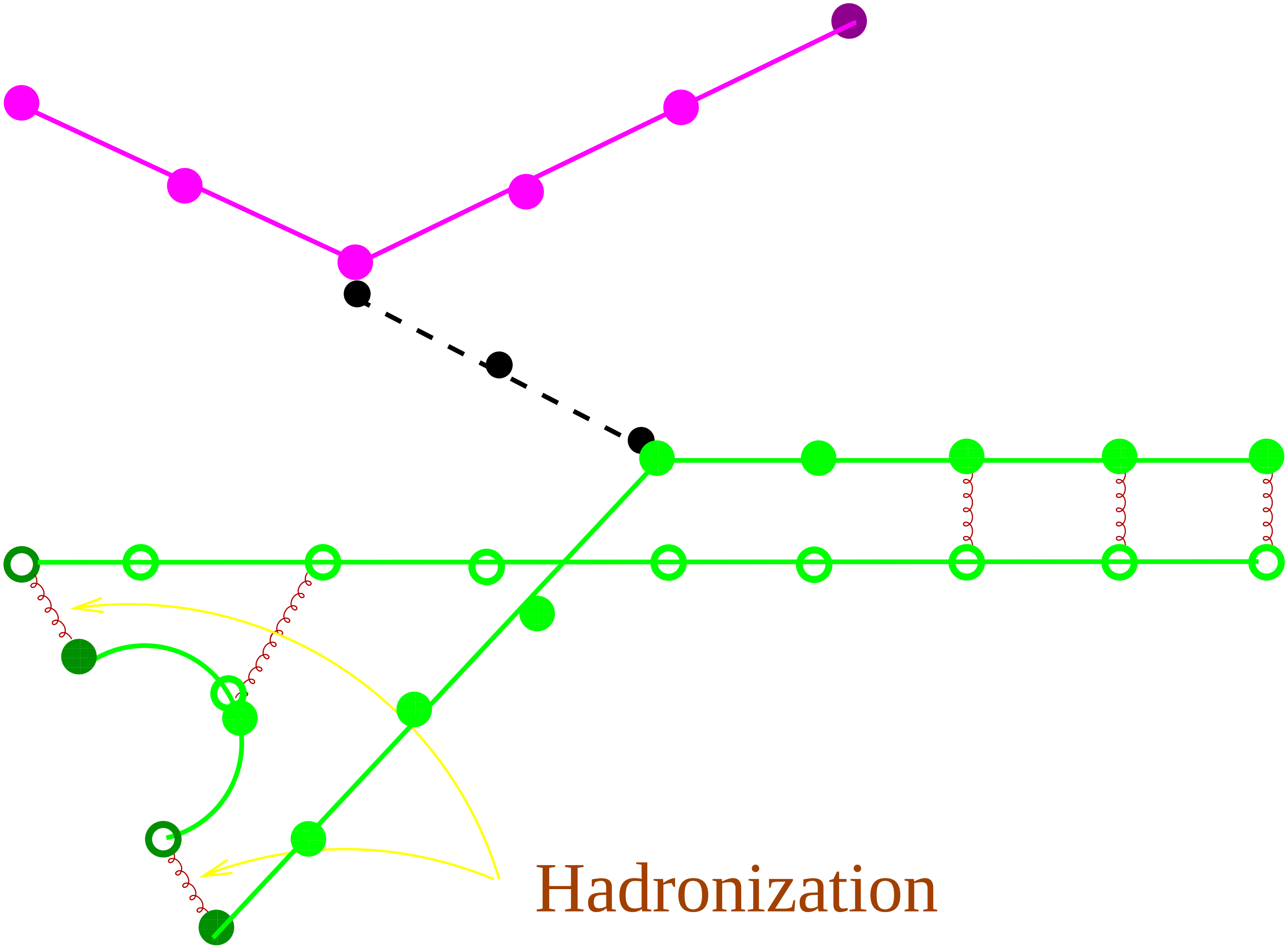


Feynman diagrams for DIS

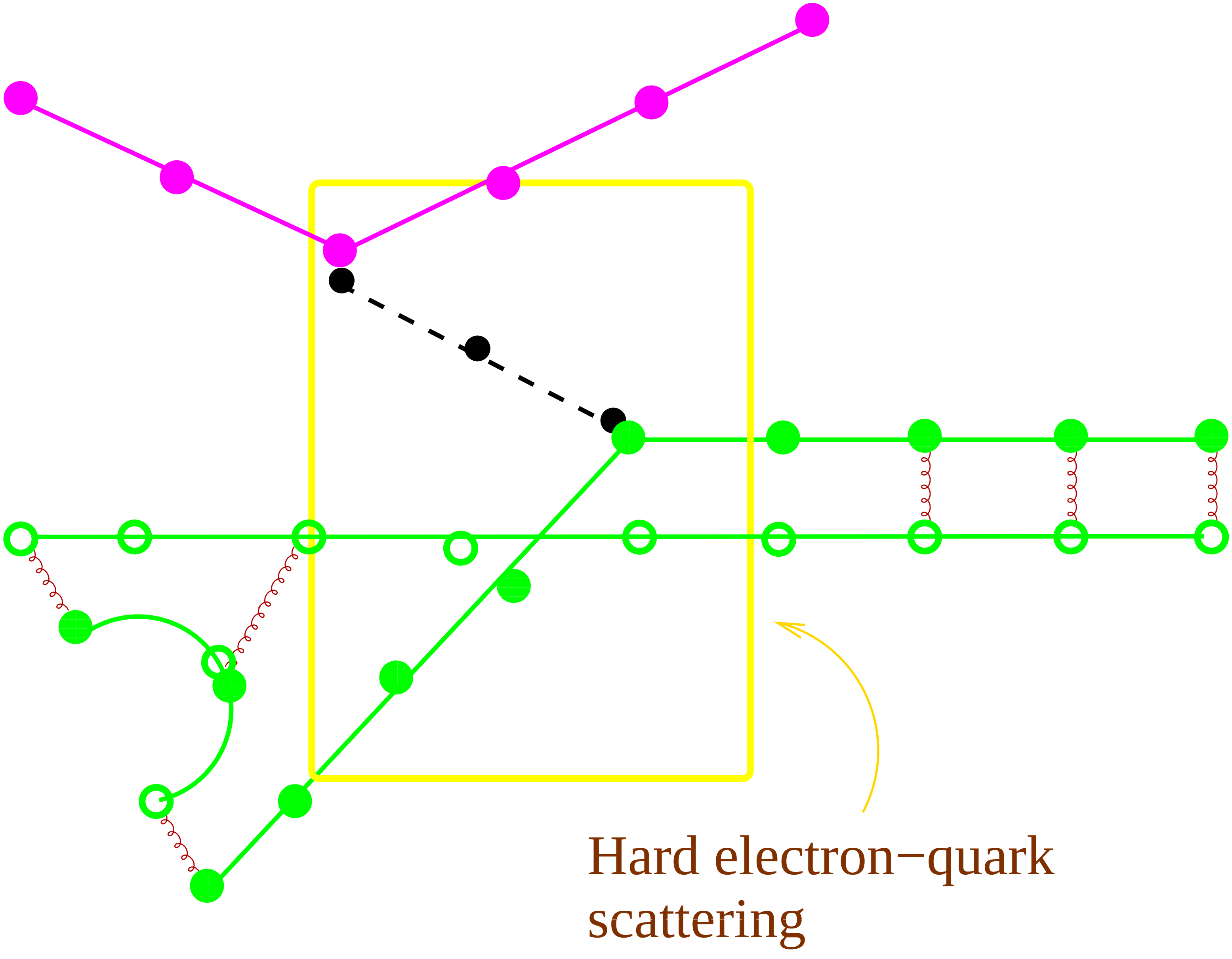



soft gluon produces
quark−antiquark pair

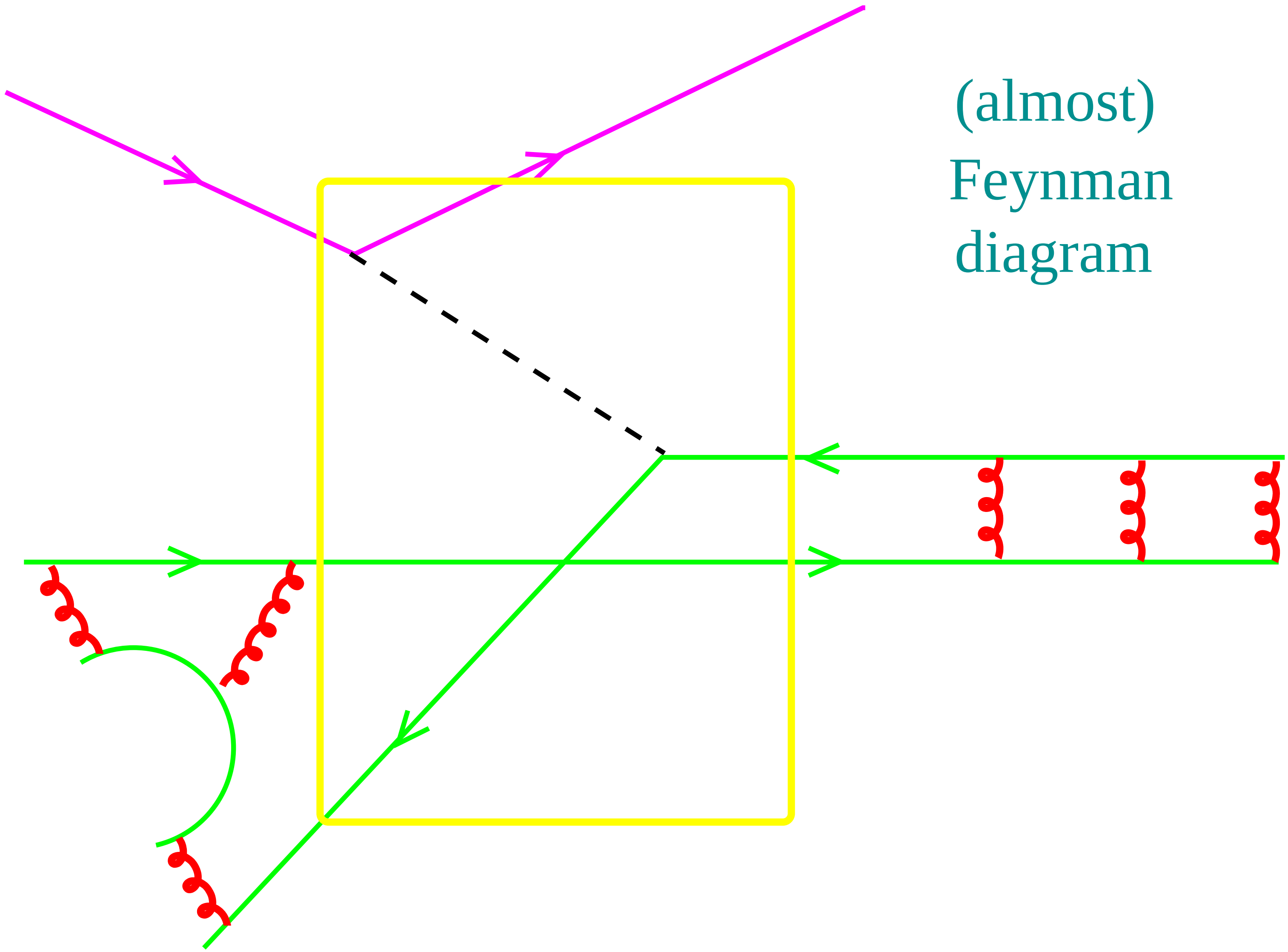
Feynman diagrams for DIS



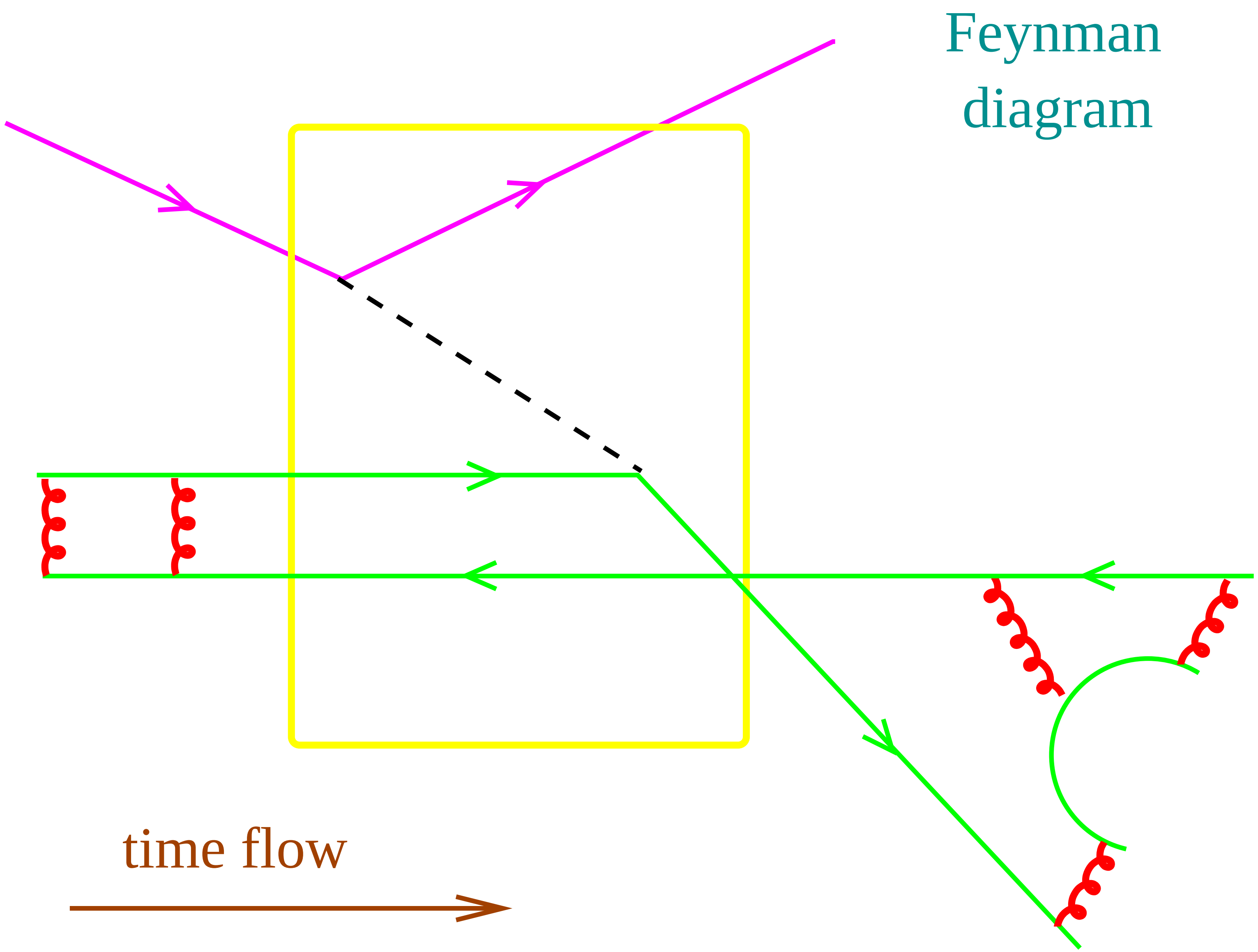
Feynman diagrams for DIS



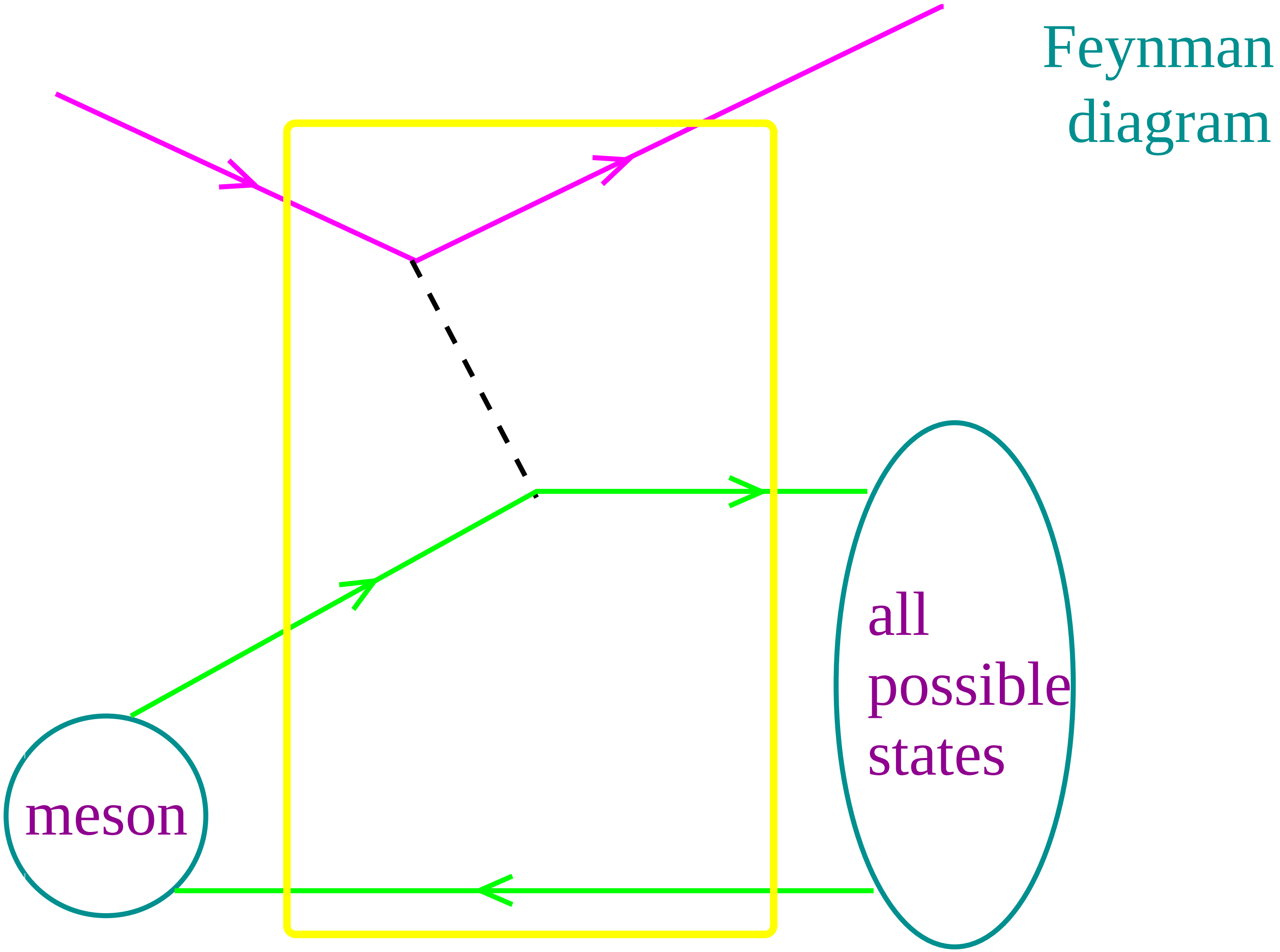
Feynman diagrams for DIS

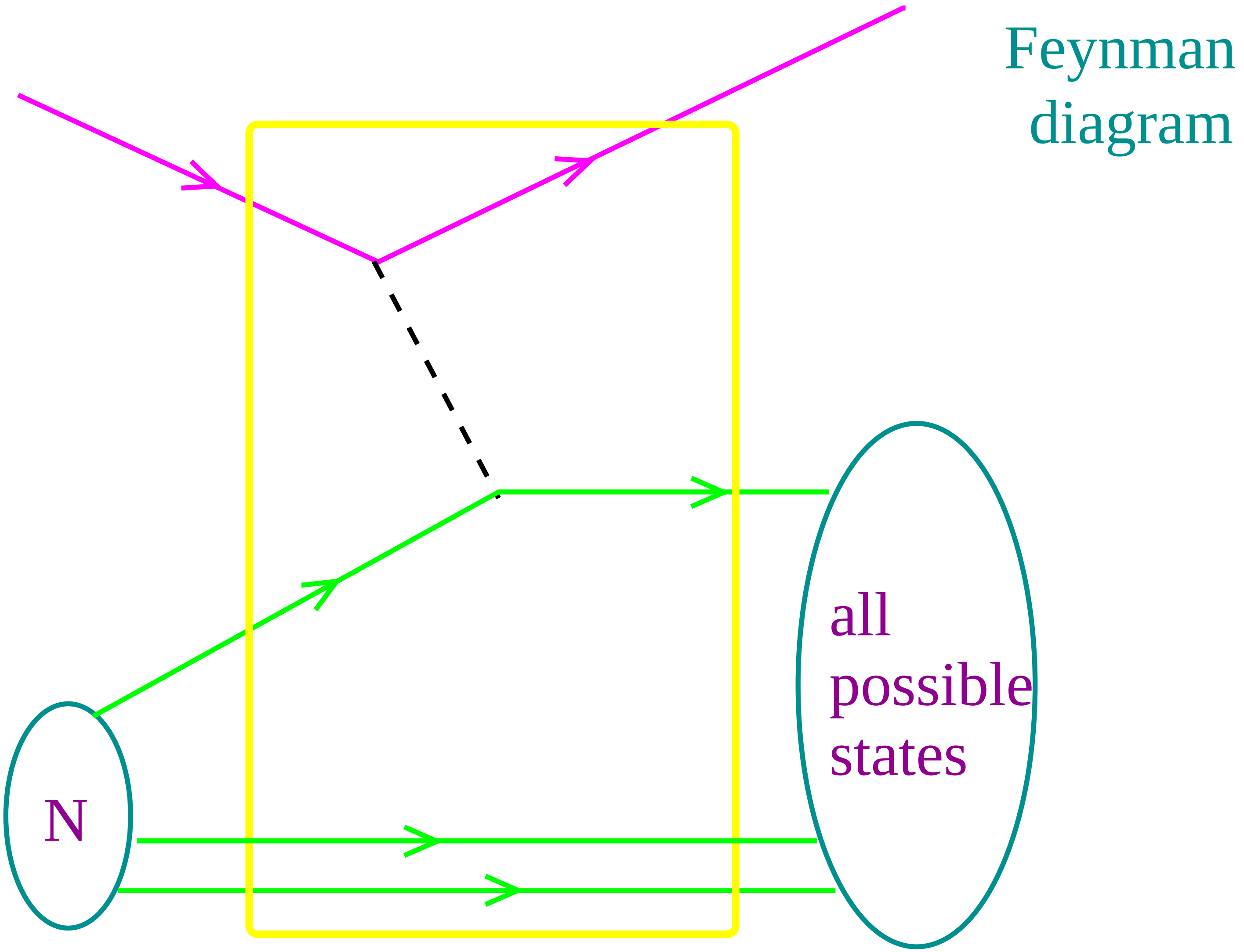


(almost)
Feynman
diagram

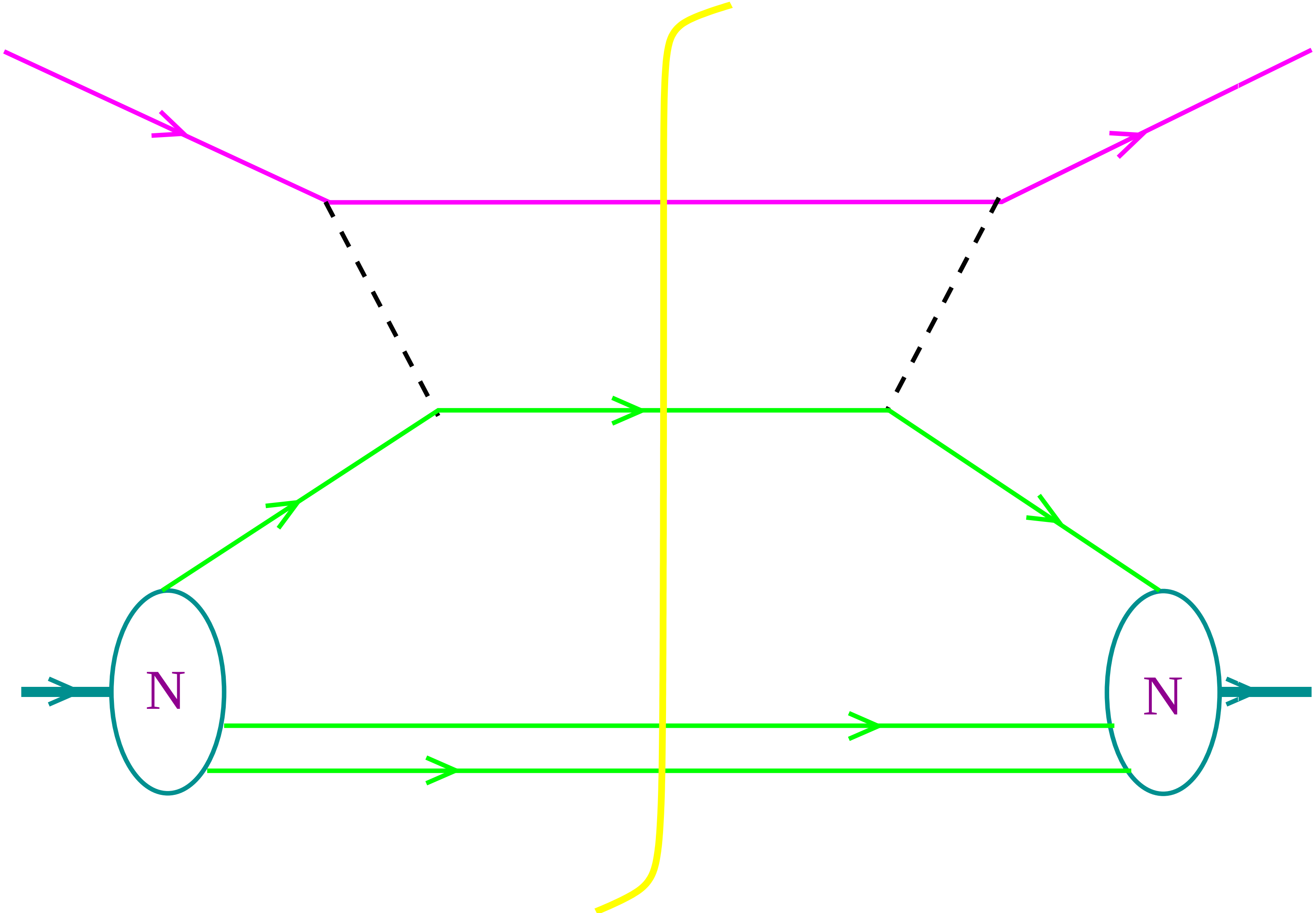

Feynman
diagram
time flow

Feynman diagrams for DIS



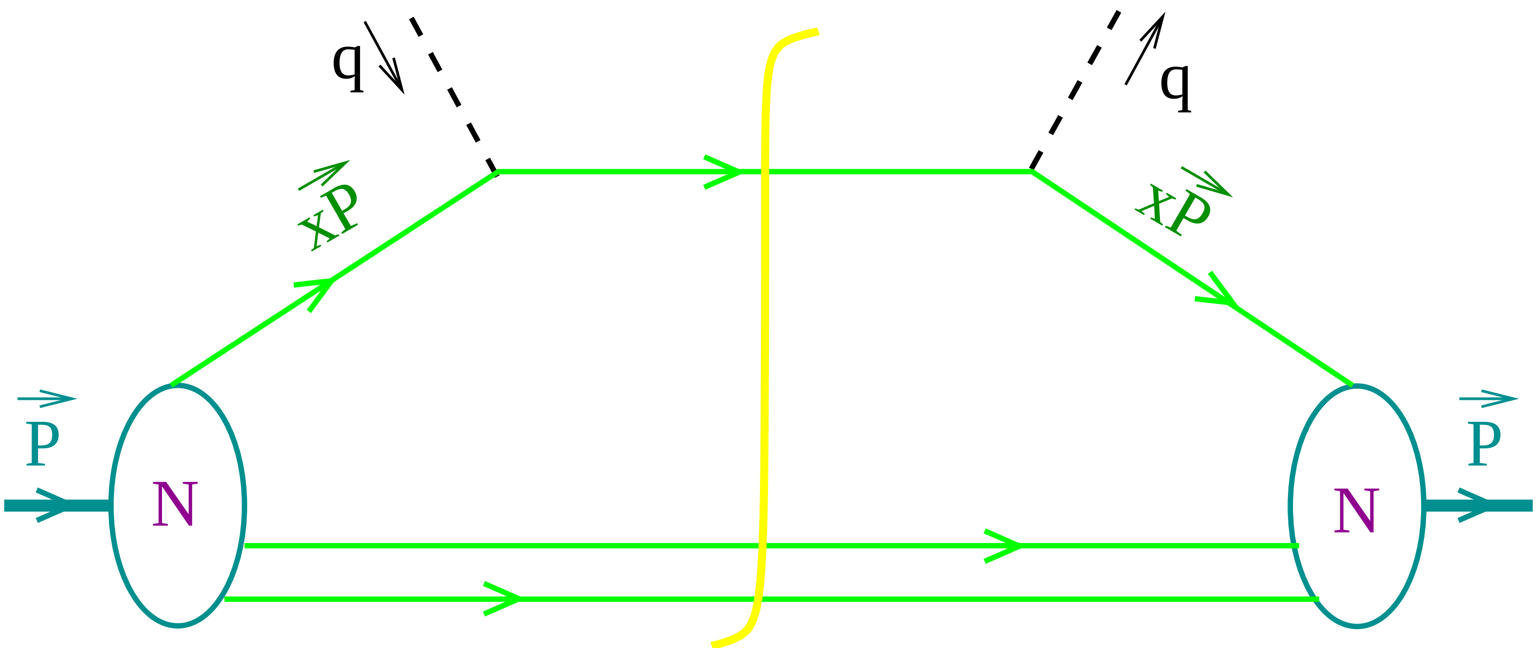

Feynman
diagram
N
all
possible
states

Optical theorem: $\sigma_{\rm tot} = \Im A_{\rm forward}$



DIS cross section in the parton model: Bjorken scaling

$$\sigma_{\text{tot}} \sim \int d^4x \, e^{iq\cdot x} \langle N | j_\mu(x) j_\nu(0) | N \rangle$$

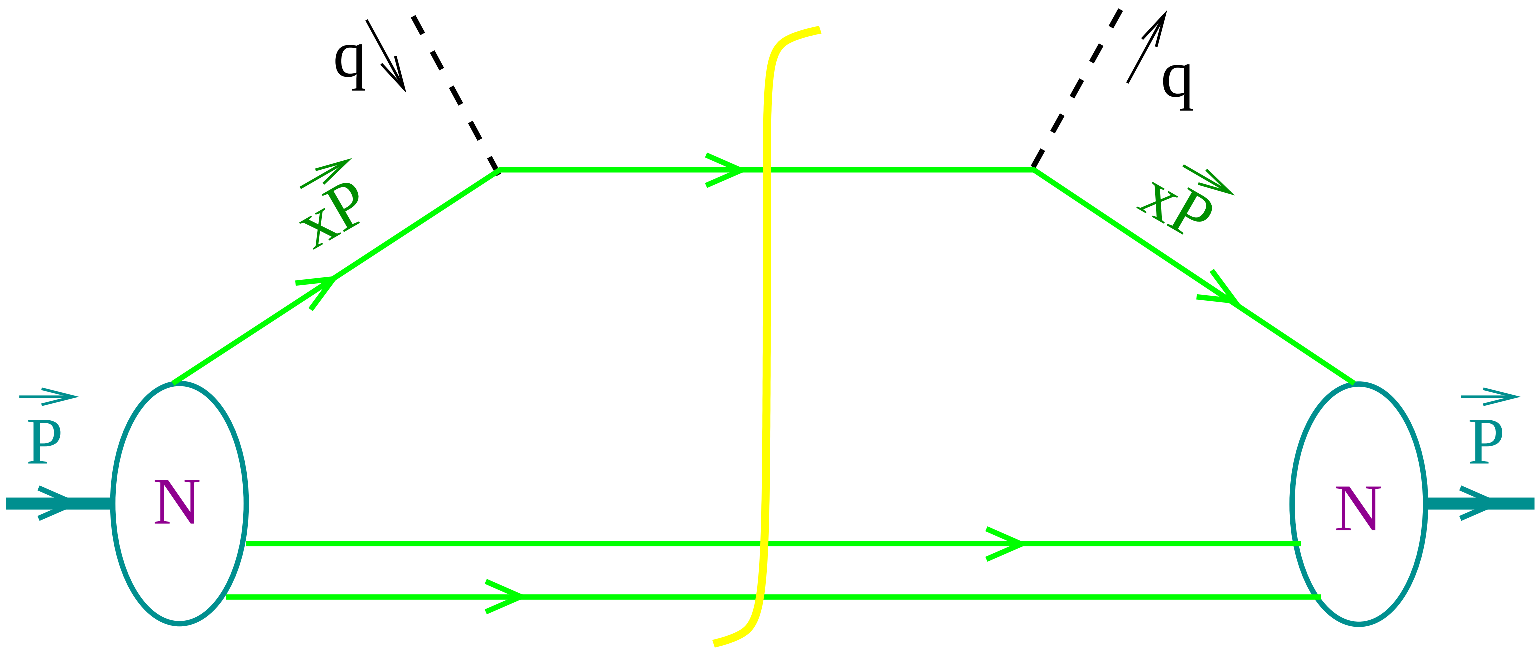


DIS cross section in the parton model: Bjorken scaling

$$\sigma_{\rm tot} \sim \int d^4x \, e^{iq\cdot x} \langle N | j_\mu(x) j_\nu(0) | N \rangle$$

Parton model (leading order of pQCD):

$$\sigma_{\rm tot} \sim \sum_q e_q^2 D_q(x_B), \qquad x_B = \frac{Q^2}{2p\cdot q}, \quad q^2 = -Q^2$$



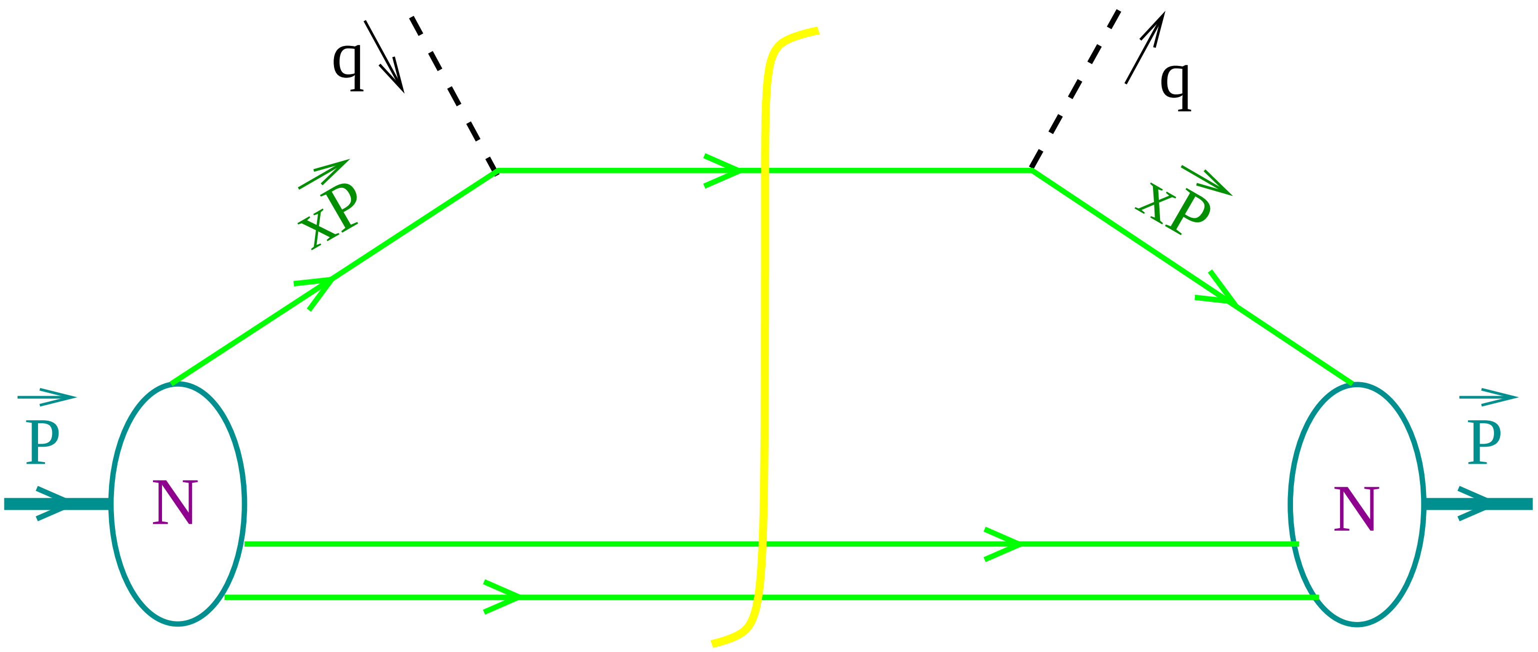
DIS cross section in the parton model: Bjorken scaling

$$\sigma_{\rm tot} \sim \int d^4x \, e^{iq\cdot x} \langle N | j_\mu(x) j_\nu(0) | N \rangle$$

Parton model (leading order of pQCD):

$$\sigma_{\rm tot} \sim \sum_q e_q^2 D_q(x_B), \qquad x_B = \frac{Q^2}{2p\cdot q}, \quad q^2 = -Q^2$$

$D_q(x)$ = probability to find the quark with fraction x of nucleon's momentum



$D_q(x_B) \longrightarrow D_q(x_B, Q^2)$ - "scaling violations"

DGLAP evolution

$$Q\frac{d}{dQ}D_q(x_B, Q^2) = K_{\rm DGLAP}D_q(x_B, Q^2)$$

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi, 1972-77

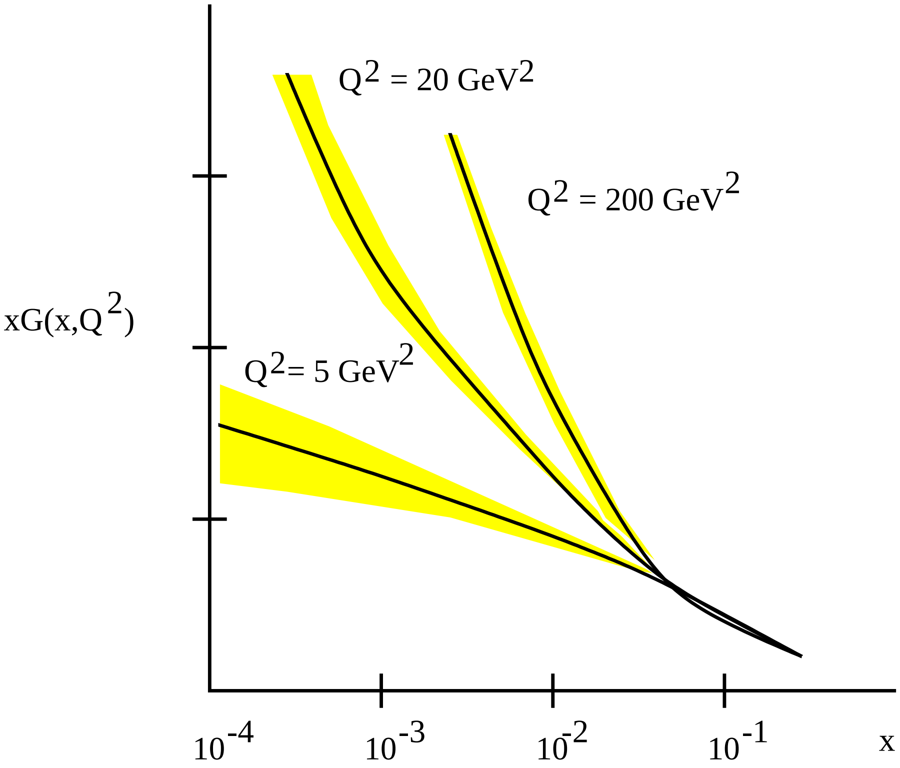
$$K_{\rm DGLAP} = \alpha_s(Q)K_{\rm LO} + \alpha_s^2(Q)K_{\rm NLO} + ...$$

One fit (at low $Q_0^2 \sim 1$ GeV^2) describes all the experimental data on DIS!

Deep inelastic scattering at small x_B

Regge limit in DIS: $E \gg Q \equiv \; x_B \ll 1$

HERA data for $xD_g(x)$

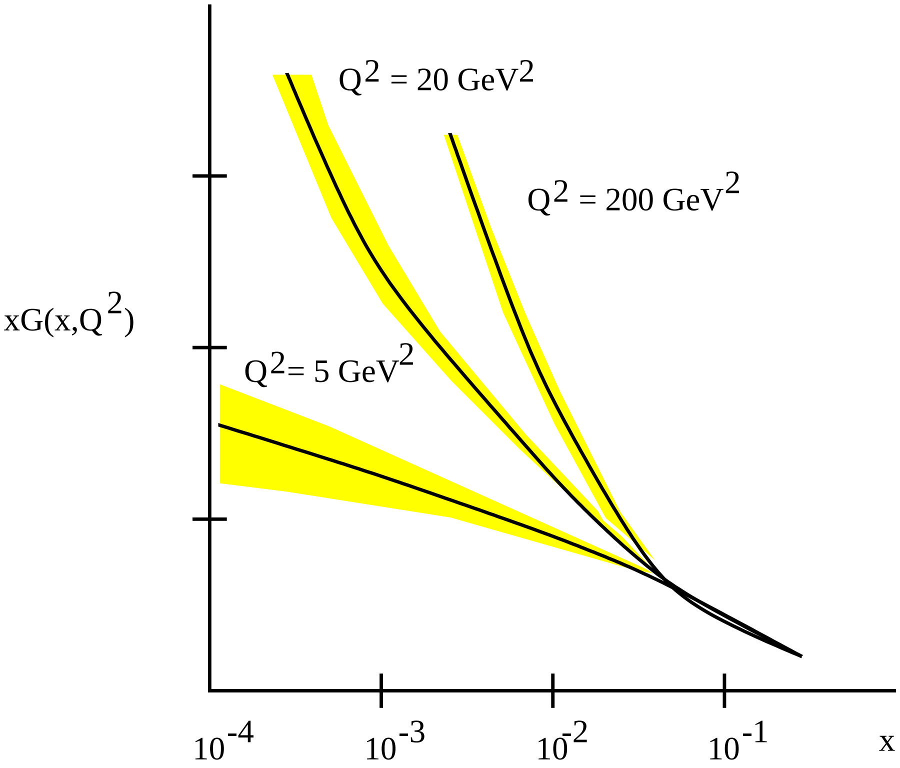


DGLAP evolution $\equiv \; Q^2$ evolution

$$Q\frac{d}{dQ}D_g(x_B, Q^2) = K_{\mathrm{DGLAP}}D_g(x_B, Q^2$$

Not really a theory -
needs the x-dependence of the input at $Q_0^2 \; \sim \; 1$GeV^2

HERA data for $xD_g(x)$



Regge limit in DIS: $E \gg Q \equiv\ x_B \ll 1$

DGLAP evolution $\equiv\ Q^2$ evolution

$$Q\frac{d}{dQ}D_g(x_B,Q^2) = K_{\mathrm{DGLAP}}D_g(x_B,Q^2)$$

Not really a theory -
needs the x-dependence of the input at $Q_0^2\ \sim\ 1\mathrm{GeV}^2$

BFKL evolution $\equiv\ x_B$ evolution
(Balitsky, Fadin, Kuraev, Lipatov, 1975-78)

$$\frac{d}{dx_B}D_g(x_B,Q^2) = K_{\mathrm{BFKL}}D_g(x_B,Q^2)$$

Theory, but with problems

High-energy scattering in QCD

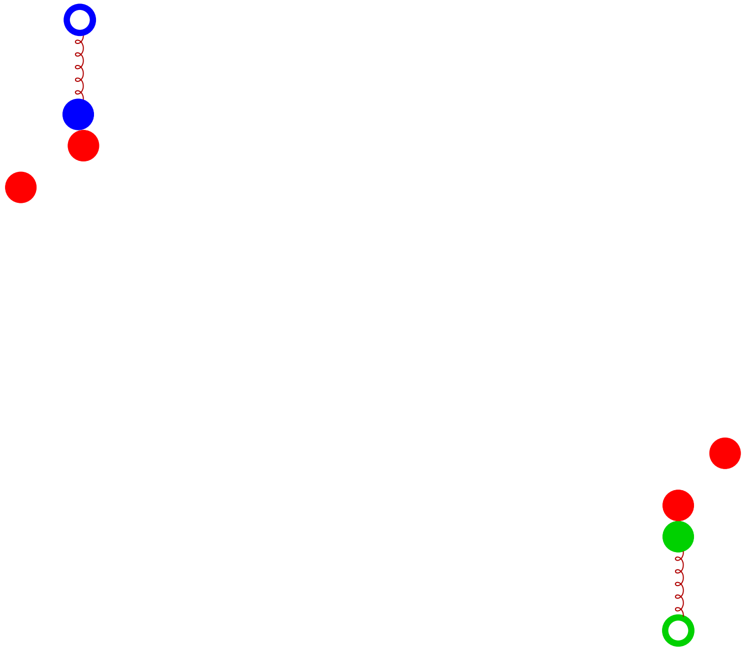
High-energy scattering in QCD

For a scattering by exchange with n particles $\sigma_{\text{tot}} \sim E^{\sum j - n}$
$\Rightarrow$ at high energy, one must have gluon exchanges

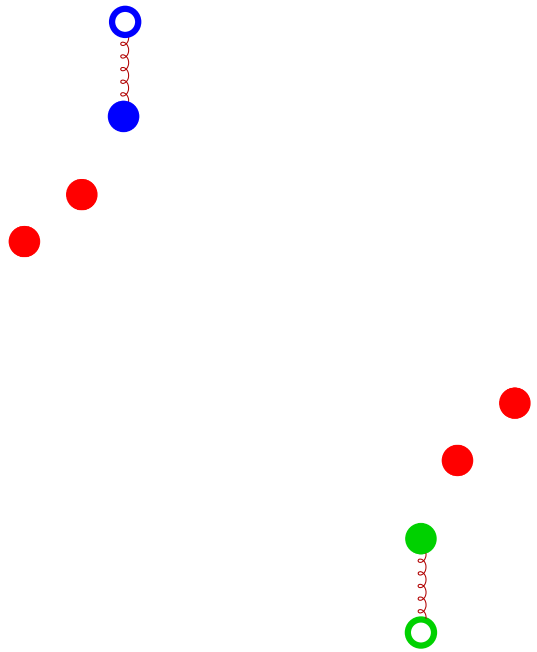
Example: 2-gluon exchange $\sigma \sim$ const; 2-quark exchange $\sigma \sim \frac{1}{s} \longrightarrow 0$

High-energy scattering in QCD

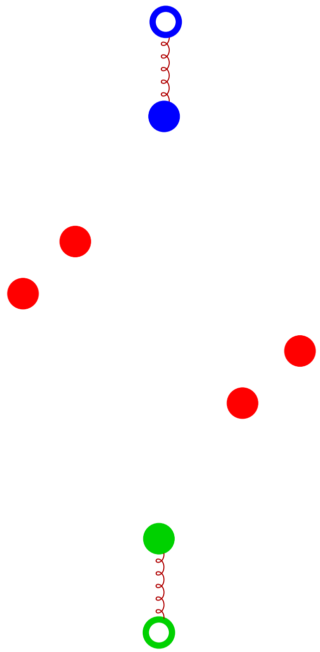
High-energy scattering in QCD

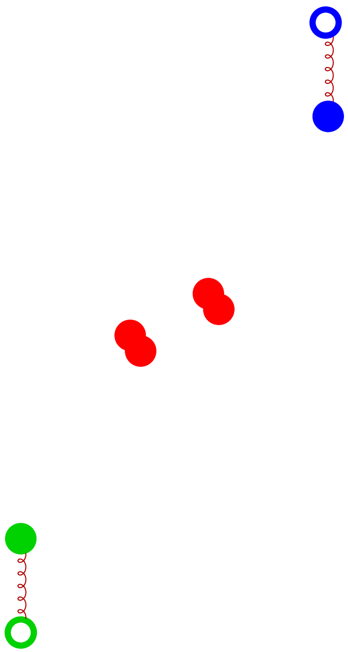


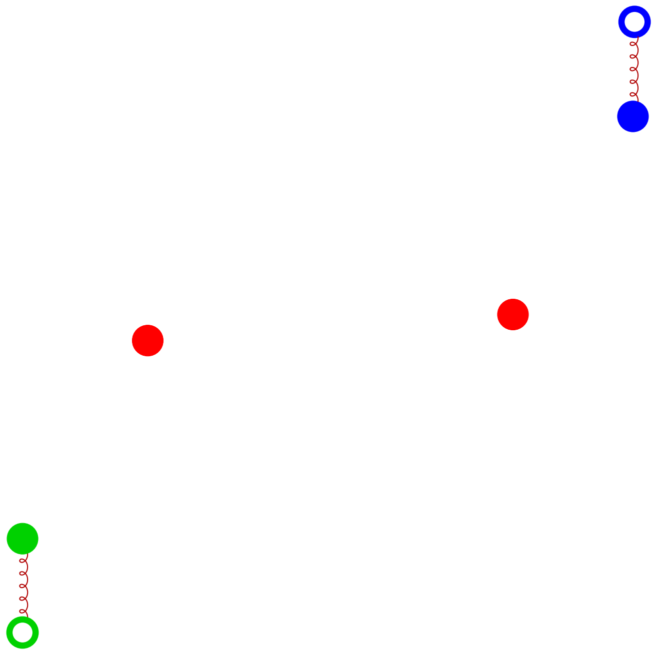
High-energy scattering in QCD



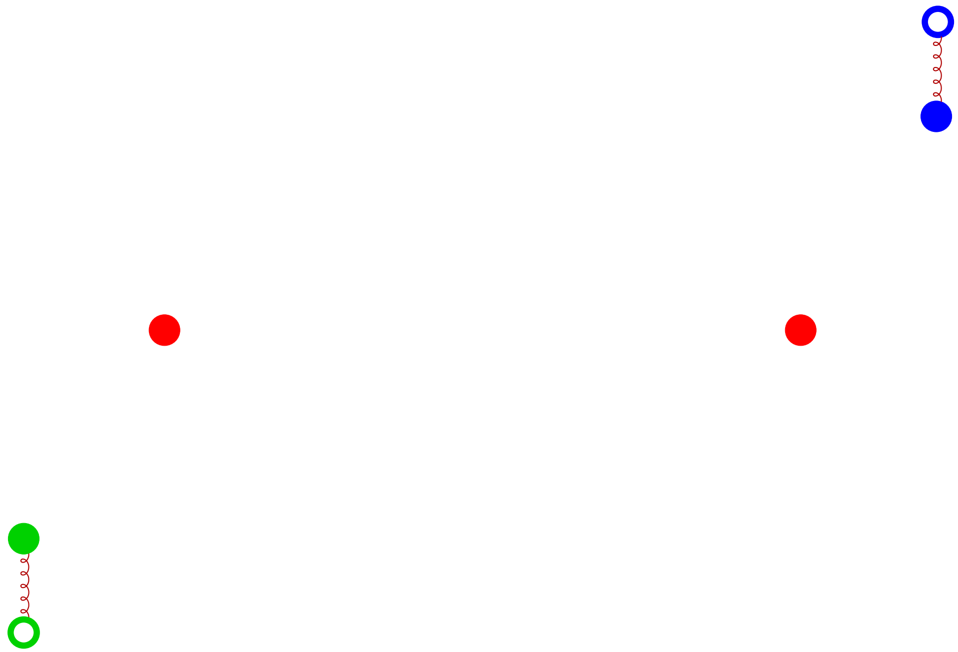
High-energy scattering in QCD







High-energy scattering in QCD

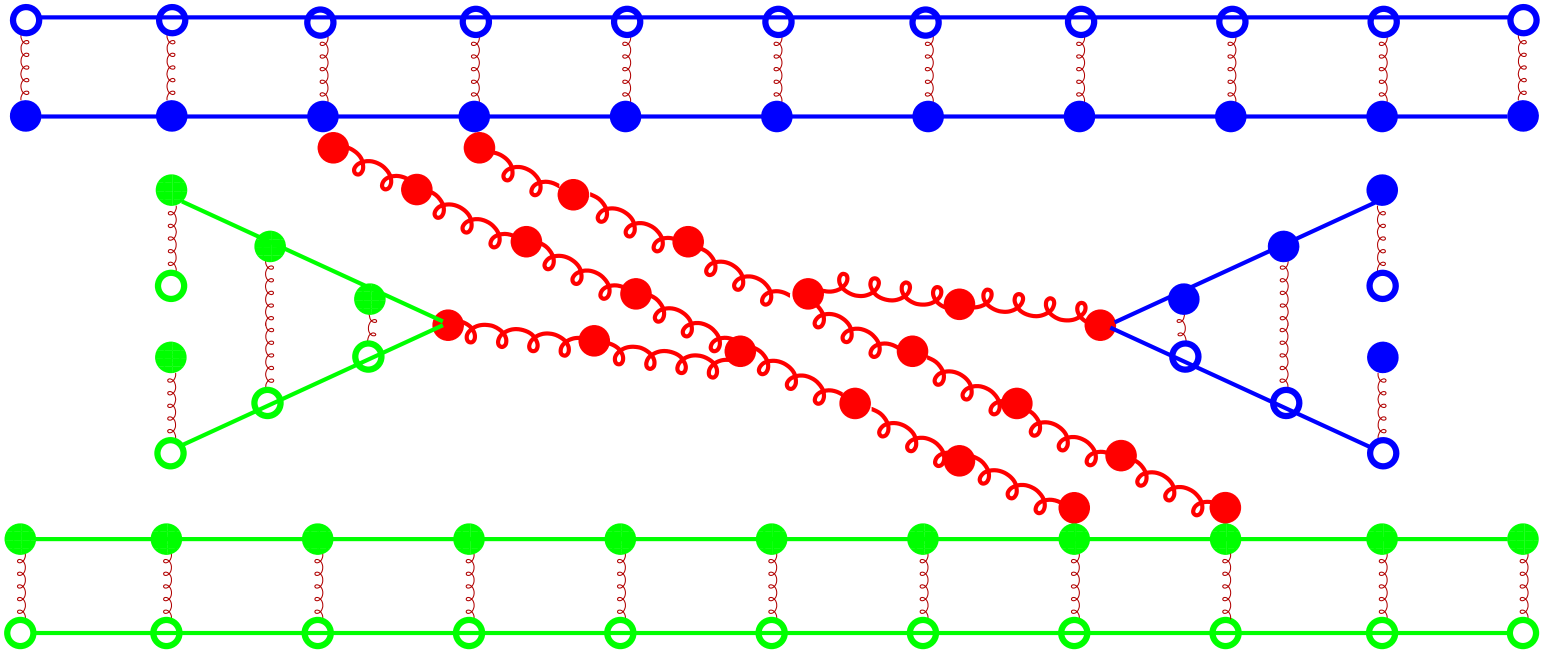


High-energy scattering in QCD

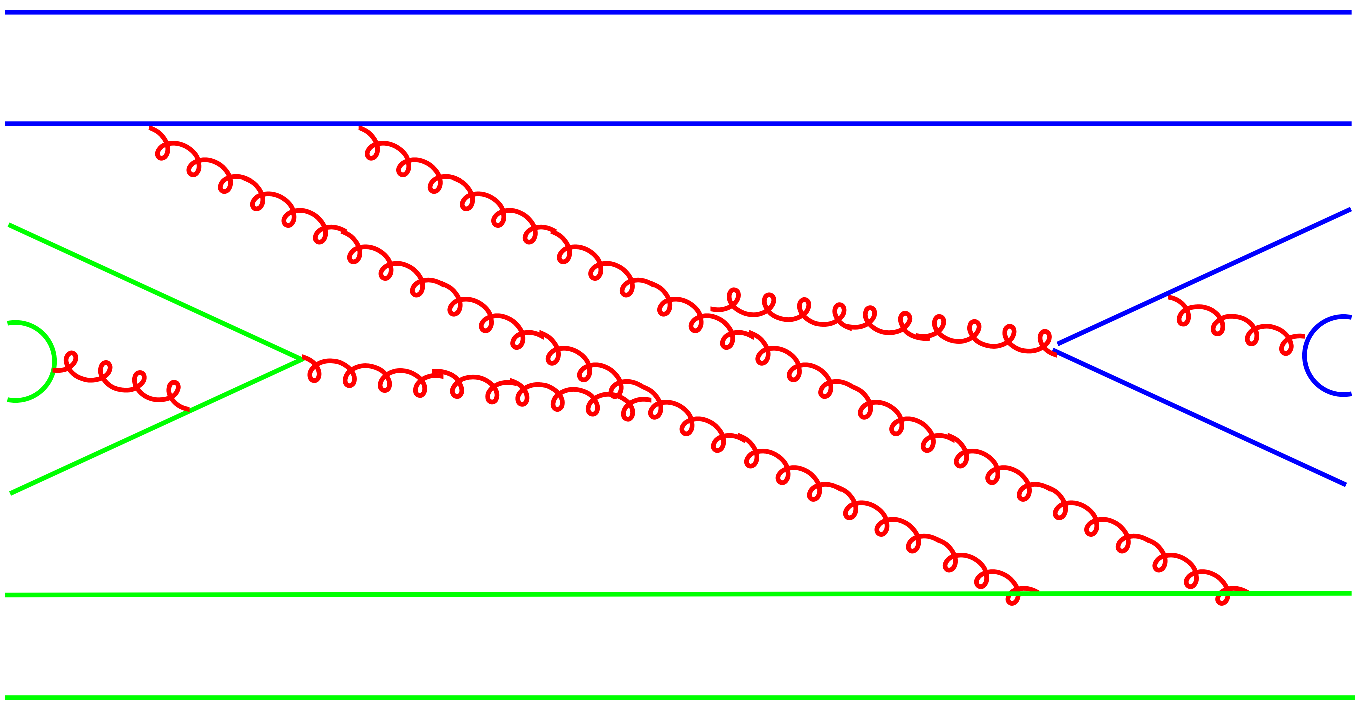
High-energy scattering in QCD

High-energy scattering in QCD

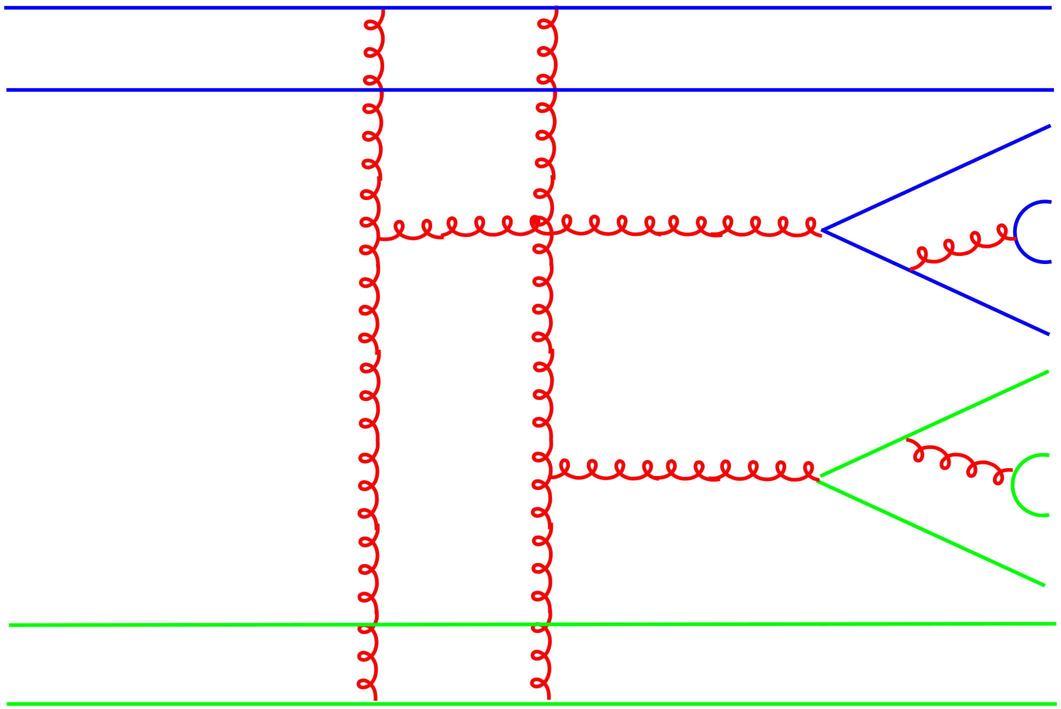
High-energy scattering in QCD



Feynman diagrams for the high-energy scattering in QCD

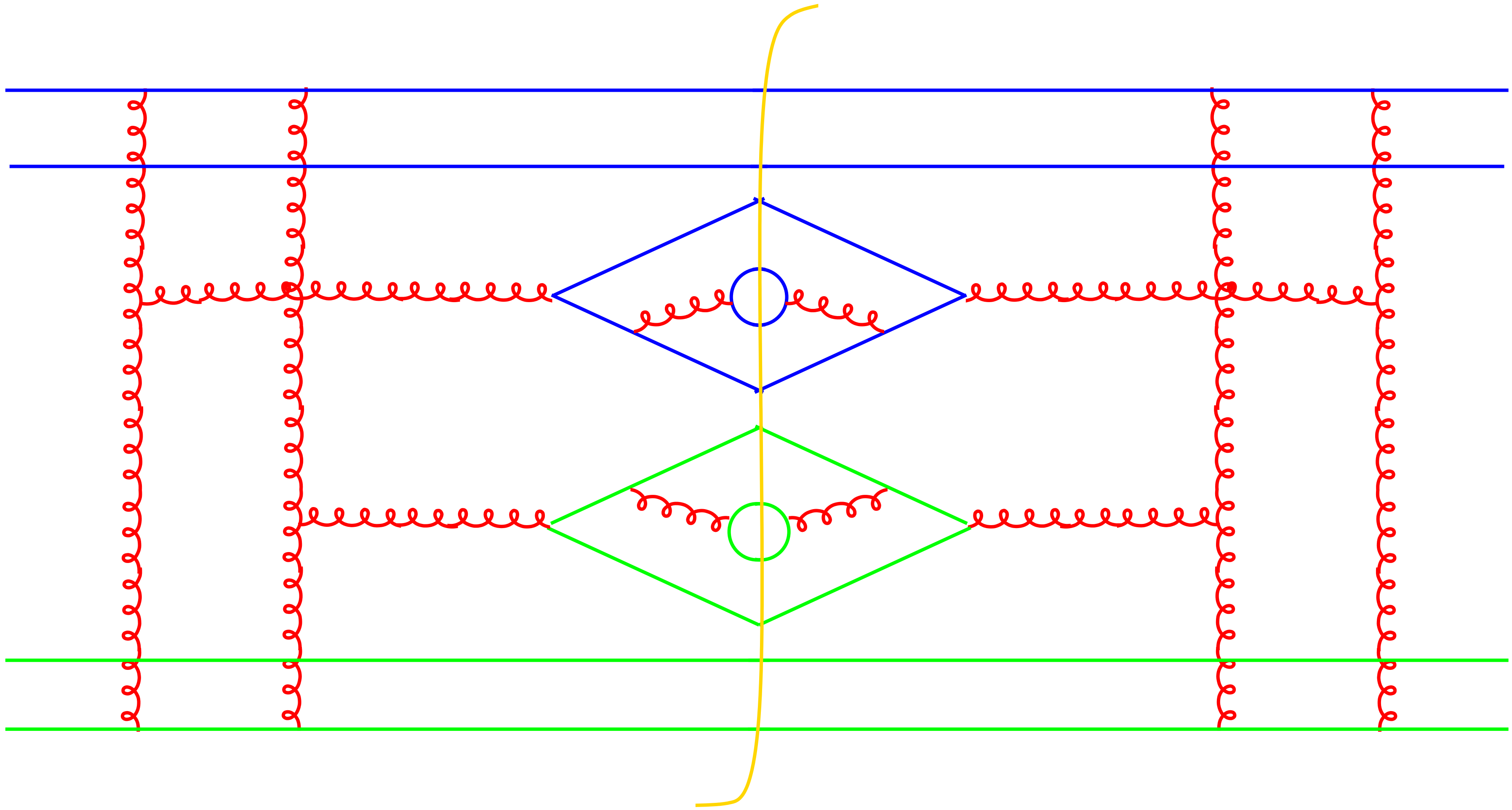


Feynman diagrams for the high-energy scattering in QCD

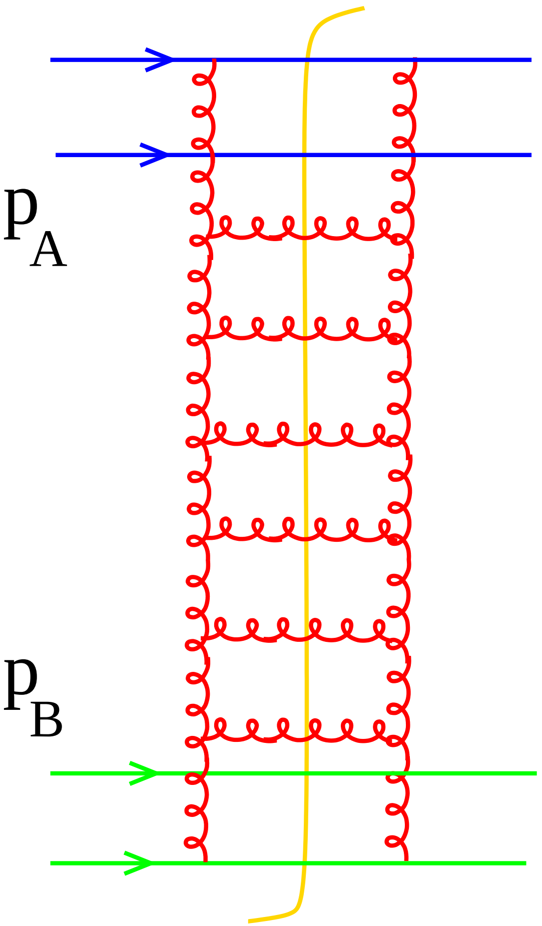


Feynman diagrams for the high-energy scattering in QCD

$\sigma_{\rm tot} \sim \Im A \Rightarrow$ diagram for the high-energy cross section



Leading Log Approximation - BFKL pomeron



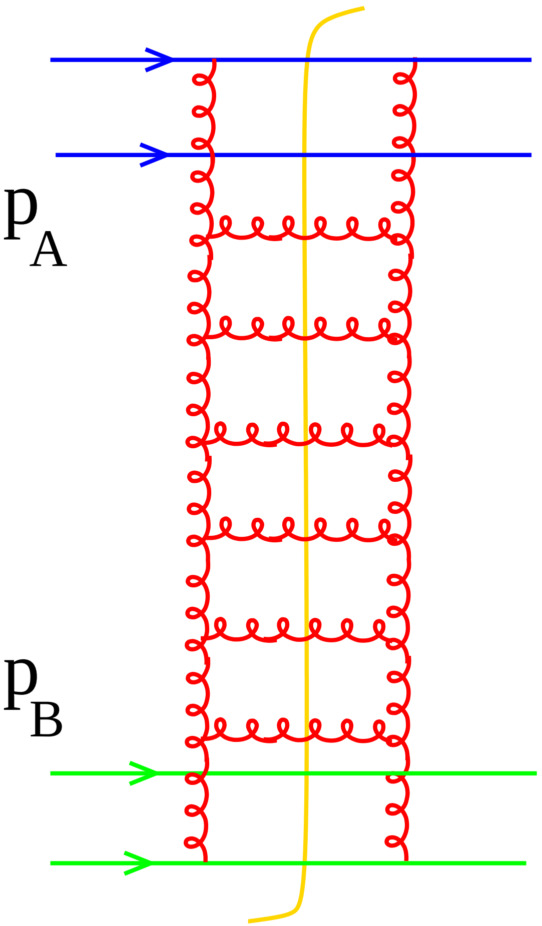
$$s = (p_A + p_B)^2 \simeq 4E^2$$

- Mandelstam variable

Leading Log Approximation (LLA):

$$\alpha_s \ll 1, \quad \alpha_s \ln s \sim 1$$

Leading Log Approximation - BFKL pomeron



$$s = (p_A + p_B)^2 \simeq 4E^2$$

- Mandelstam variable

Leading Log Approximation (LLA):

$$\alpha_s \ll 1, \quad \alpha_s \ln s \sim 1$$

The sum of gluon ladder diagrams gives

$$\sigma_{\rm tot} \sim s^{12\frac{\alpha_s}{\pi}\ln 2}$$

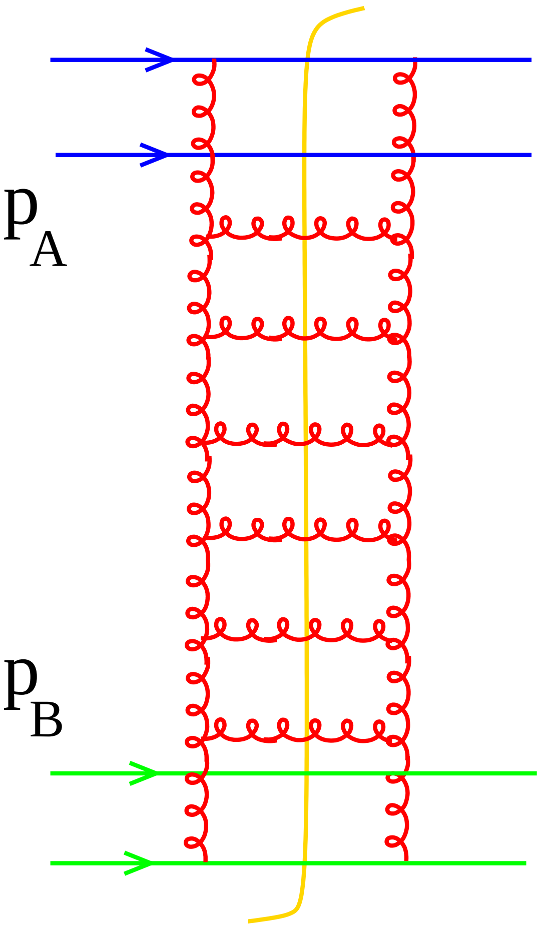
BFKL pomeron

Numerically: for DIS at HERA

$$\sigma \sim s^{0.3 \div 0.5} = x_B^{-0.3 \div 0.5}$$

- qualitatively OK

Leading Log Approximation - BFKL pomeron



$$s = (p_A + p_B)^2 \simeq 4E^2$$

- Mandelstam variable

Leading Log Approximation (LLA):

$$\alpha_s \ll 1, \quad \alpha_s \ln s \sim 1$$

The sum of gluon ladder diagrams gives

$$\sigma_{\text{tot}} \sim s^{12\frac{\alpha_s}{\pi}\ln 2}$$

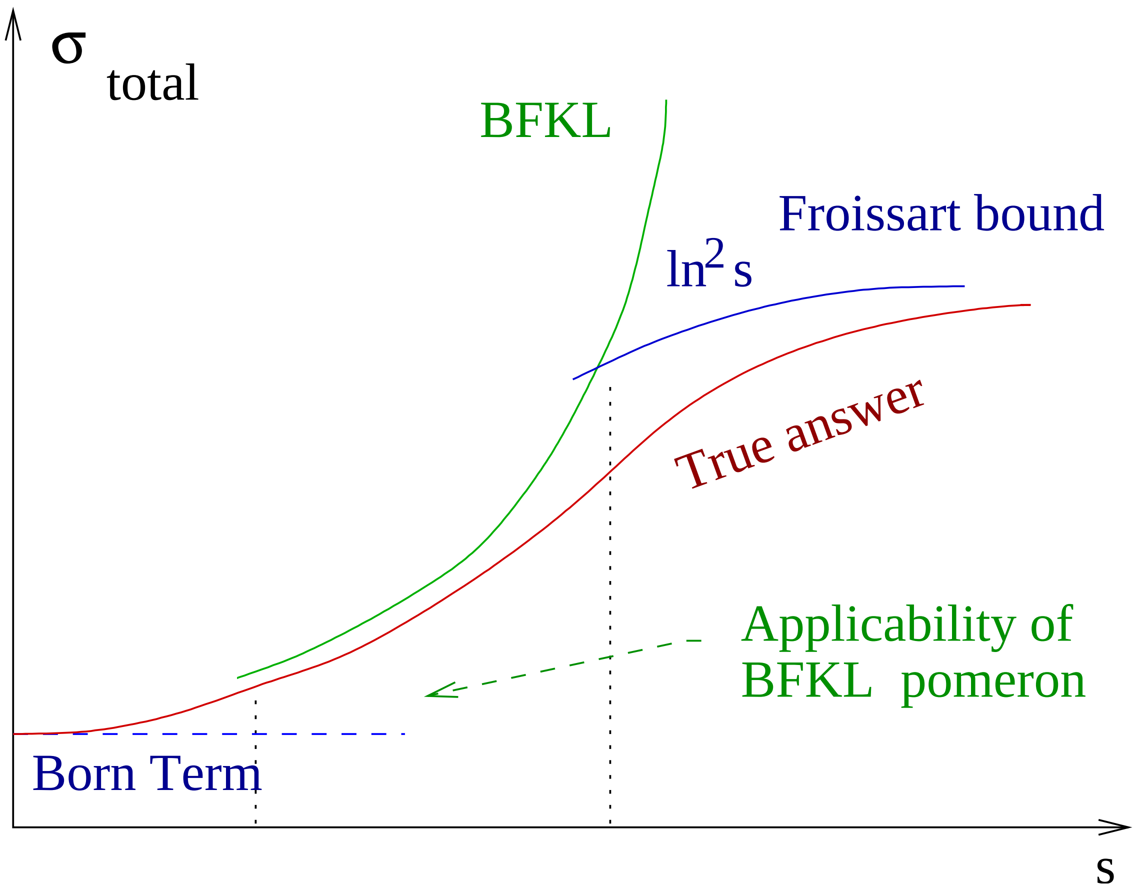
BFKL pomeron

Numerically: for DIS at HERA

$$\sigma \sim s^{0.3 \div 0.5} = x_B^{-0.3 \div 0.5}$$

- qualitatively OK

Towards the high-energy QCD

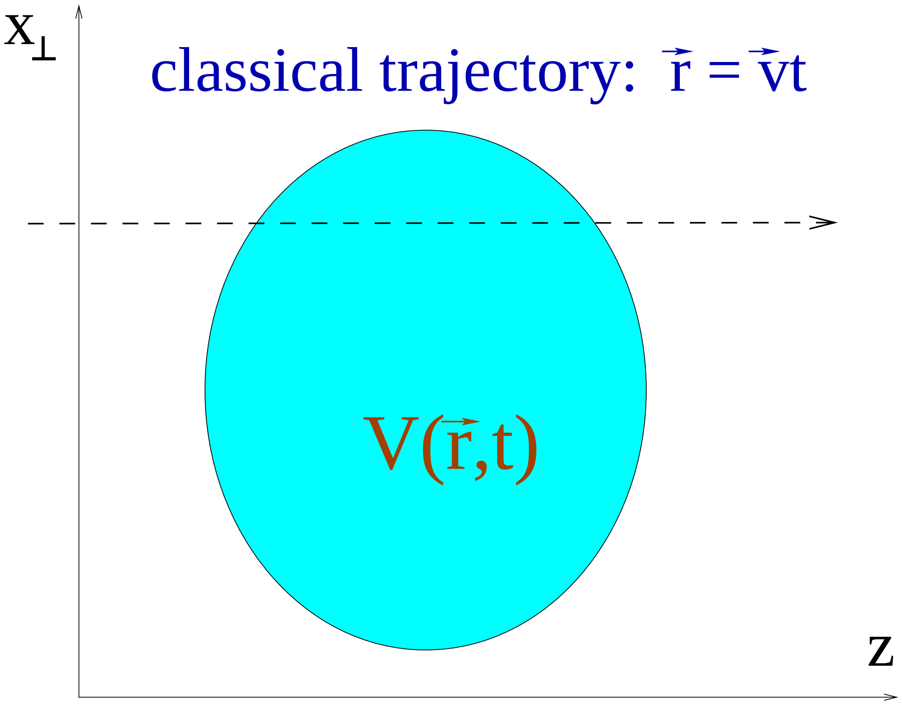


$\sigma_{\rm tot} \sim s^{12\frac{\alpha_s}{\pi}\ln 2}$ violates
Froissart bound $\sigma_{\rm tot} \leq \ln^2 s$
$\Rightarrow$ pre-asymptotic behavior.

True asymptotics as $E \to \infty$ = ?
Possible approaches:

- Sum all logs $\alpha_s^m \ln^n s$
- Reduce high-energy QCD to the effective theory in $2+1$ dimensions

High-energy scattering in quantum mechanics

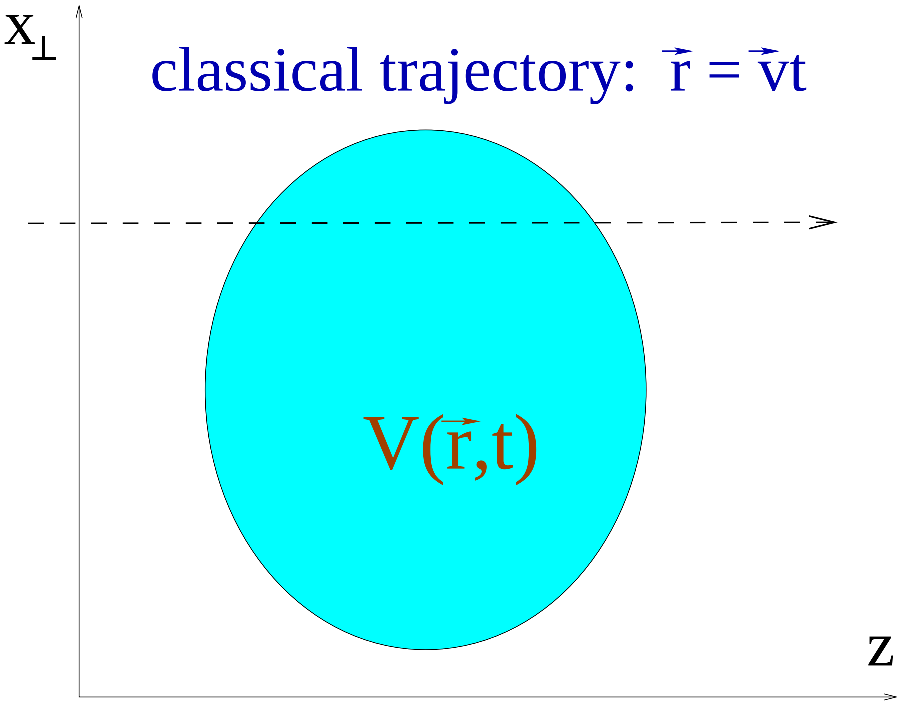


WKB approximation: $\Psi \sim e^{\frac{i}{\hbar}S}$

$$S = \int (pdz - Edt)$$

$$= -Et + \int^{z} dz' \sqrt{2m(E - V(z')}$$

High-energy scattering in quantum mechanics



WKB approximation: $\Psi \sim e^{\frac{i}{\hbar}S}$

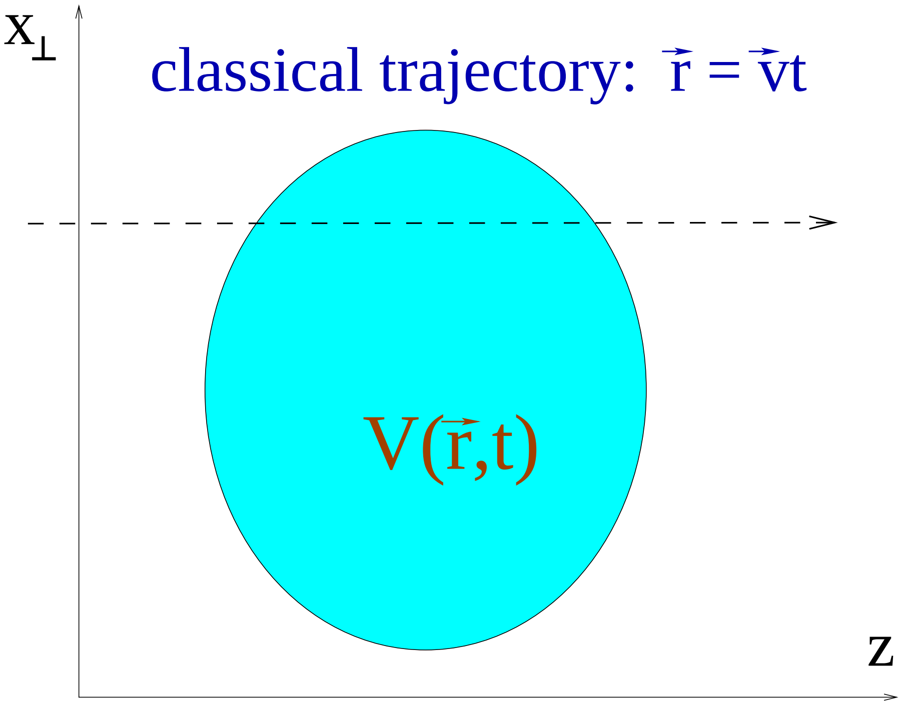
$$S = \int (pdz - Edt)$$

$$= -Et + \int^z dz' \sqrt{2m(E - V(z')}$$

High energy: $E \gg V(x) \Rightarrow$

$$\Psi(\vec{r},t) = e^{-\frac{i}{\hbar}(Et - kx)}\, e^{-\frac{i}{v\hbar}\int_{-\infty}^{z} dz' V(z')}$$

High-energy scattering in quantum mechanics



WKB approximation: $\Psi \sim e^{\frac{i}{\hbar}S}$

$$S = \int (pdz - Edt)$$

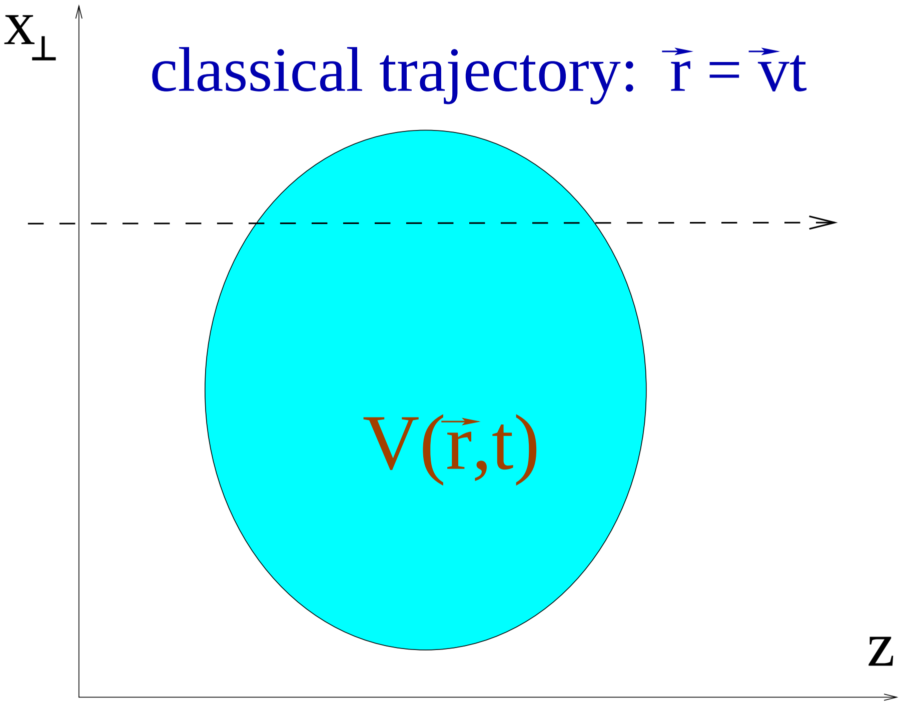
$$= -Et + \int^z dz' \sqrt{2m(E - V(z'))}$$

High energy: $E \gg V(x) \Rightarrow$

$$\Psi(\vec{r}, t) = e^{-\frac{i}{\hbar}(Et - kx)} \; e^{-\frac{i}{v\hbar}\int_{-\infty}^{z} dz' V(z')}$$

Ψ at high energy = free wave $\times$ phase factor ordered along the line $\parallel$ $\vec{v}$.

High-energy scattering in quantum mechanics



WKB approximation: $\Psi \sim e^{\frac{i}{\hbar}S}$

$$S = \int (p dz - E dt)$$

$$= -Et + \int^z dz' \sqrt{2m(E - V(z')}$$

High energy: $E \gg V(x) \Rightarrow$

$$\Psi(\vec{r}, t) = e^{-\frac{i}{\hbar}(Et - kx)}\, e^{-\frac{i}{v\hbar}\int_{-\infty}^{z} dz' V(z')}$$

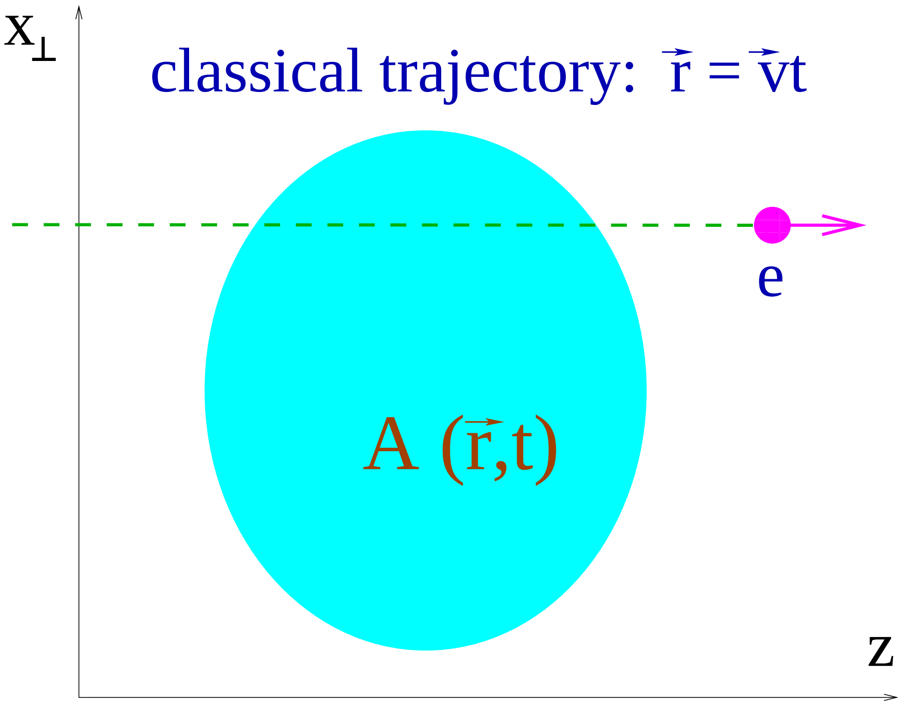
Ψ at high energy = free wave $\times$ phase factor ordered along the line $\parallel \vec{v}$.

The scattering amplitude is proportional to $\Psi(t = \infty)$ defined by

$$U(x_\perp) \ = \ e^{-\frac{i}{v\hbar}\int_{-\infty}^{\infty} dz' V(z' + x_\perp)}$$

Glauber formula: $\sigma_{\rm tot} = 2\int d^2x_\perp \left[1 - \Re U(x_\perp)\right]$

High-energy phase factor in QED and QCD

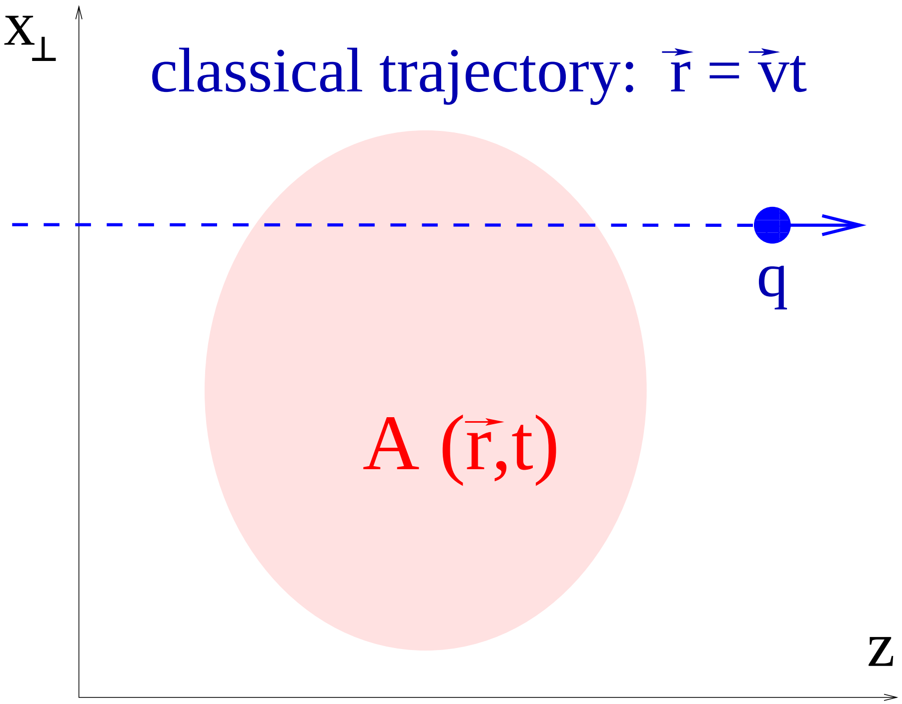


$$S_e = \int dt \left\{ -mc^2\sqrt{1-\frac{v^2}{c^2}} - e\Phi + \frac{e}{c}\vec{v}\cdot\vec{A} \right\}$$
$$= S_{\rm free} + \int dt(-e\Phi + \frac{e}{c}\vec{v}\cdot\vec{A})$$

$\Rightarrow$ phase factor for the high-energy scattering is

$$U(x_\perp, v) = e^{-\frac{ie}{\hbar c}\int_{-\infty}^{\infty} dt(-e\Phi+\frac{e}{c}\vec{v}\cdot\vec{A})}$$
$$= e^{-\frac{ie}{\hbar c}\int_{-\infty}^{\infty} dt\ \dot{x}_\mu A^\mu(x(t))}$$

High-energy phase factor in QED and QCD



$$S_e = \int dt \left\{ - mc^2 \sqrt{1 - \frac{v^2}{c^2}} - e\Phi + \frac{e}{c}\vec{v}\cdot\vec{A} \right\}$$

$$= S_{\text{free}} + \int dt(-e\Phi + \frac{e}{c}\vec{v}\cdot\vec{A})$$

$\Rightarrow$ phase factor for the high-energy scattering is

$$U(x_\perp, v) = e^{-\frac{ie}{\hbar c}\int_{-\infty}^{\infty} dt(-e\Phi + \frac{e}{c}\vec{v}\cdot\vec{A})}$$

$$= e^{-\frac{ie}{\hbar c}\int_{-\infty}^{\infty} dt\ \dot{x}_\mu A^\mu(x(t))}$$

In QCD $e \to g$, $A_\mu \to A_\mu \equiv A^a_\mu \frac{\lambda^a}{2}$

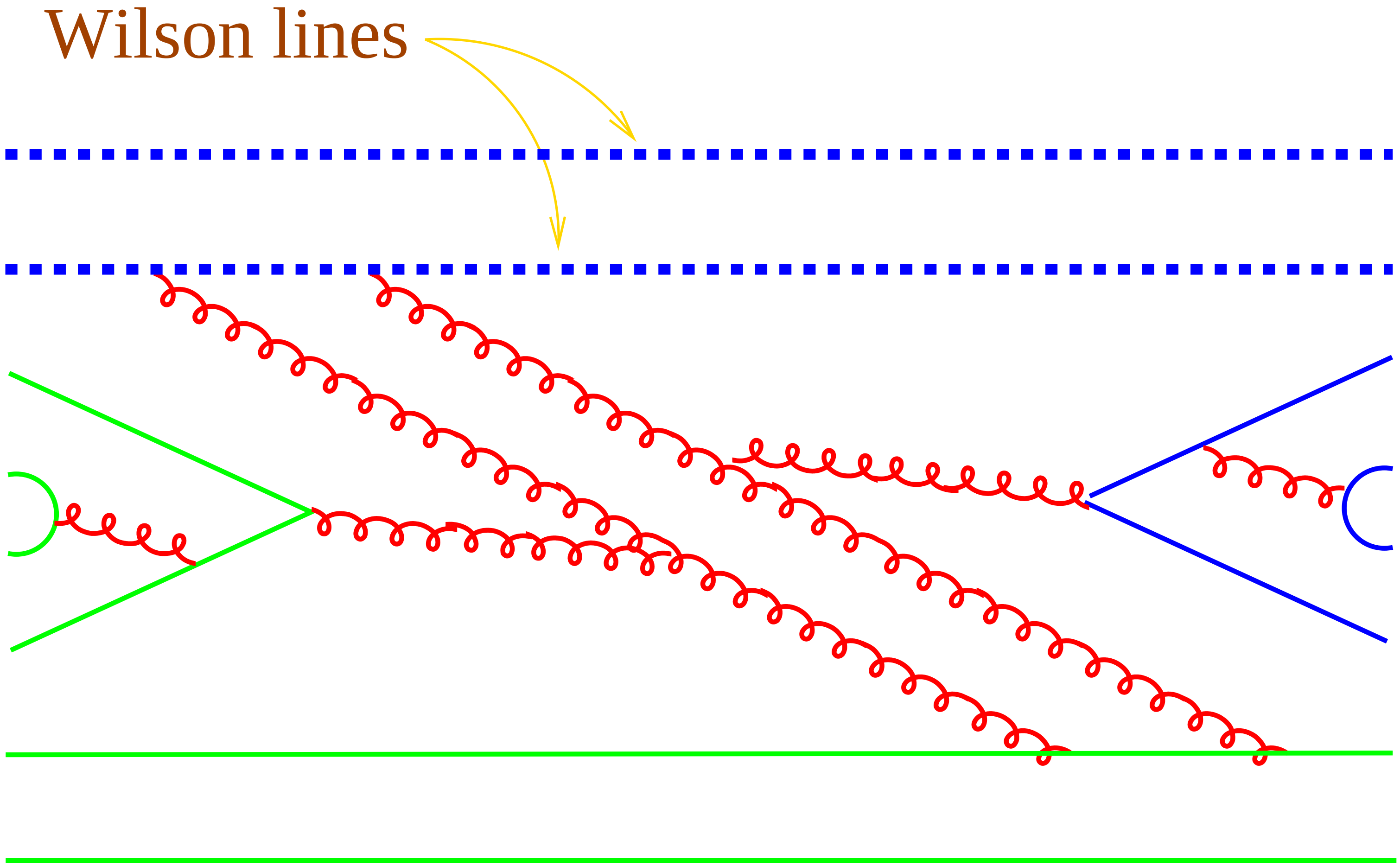
λ^a - Gell-Mann matrices

$$\Rightarrow\ U(x_\perp, v) = Pe^{-\frac{ie}{\hbar c}\int_{-\infty}^{\infty} dt\ \dot{x}_\mu A^\mu(x(t))}$$

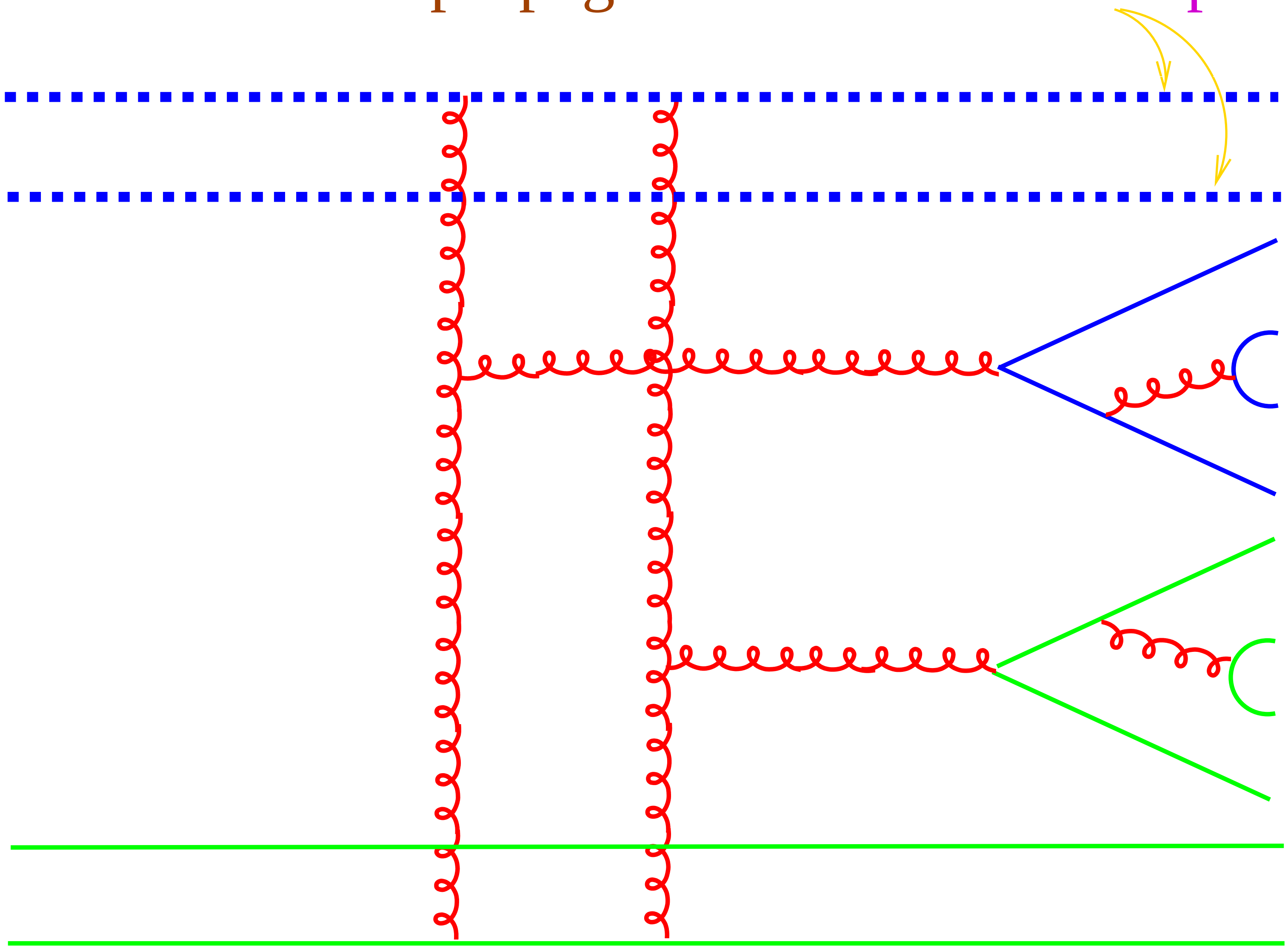
$\mathrm{Wilson-line\ \ operator}$

$$Pe^{\int_a^b dt\ \mathcal{O}(t)} \equiv 1 + \int_a^b dt\mathcal{O}(t) + \int_a^b dt\mathcal{O}(t)\int_a^t dt'\mathcal{O}(t') + ...$$

Wilson lines and color dipoles



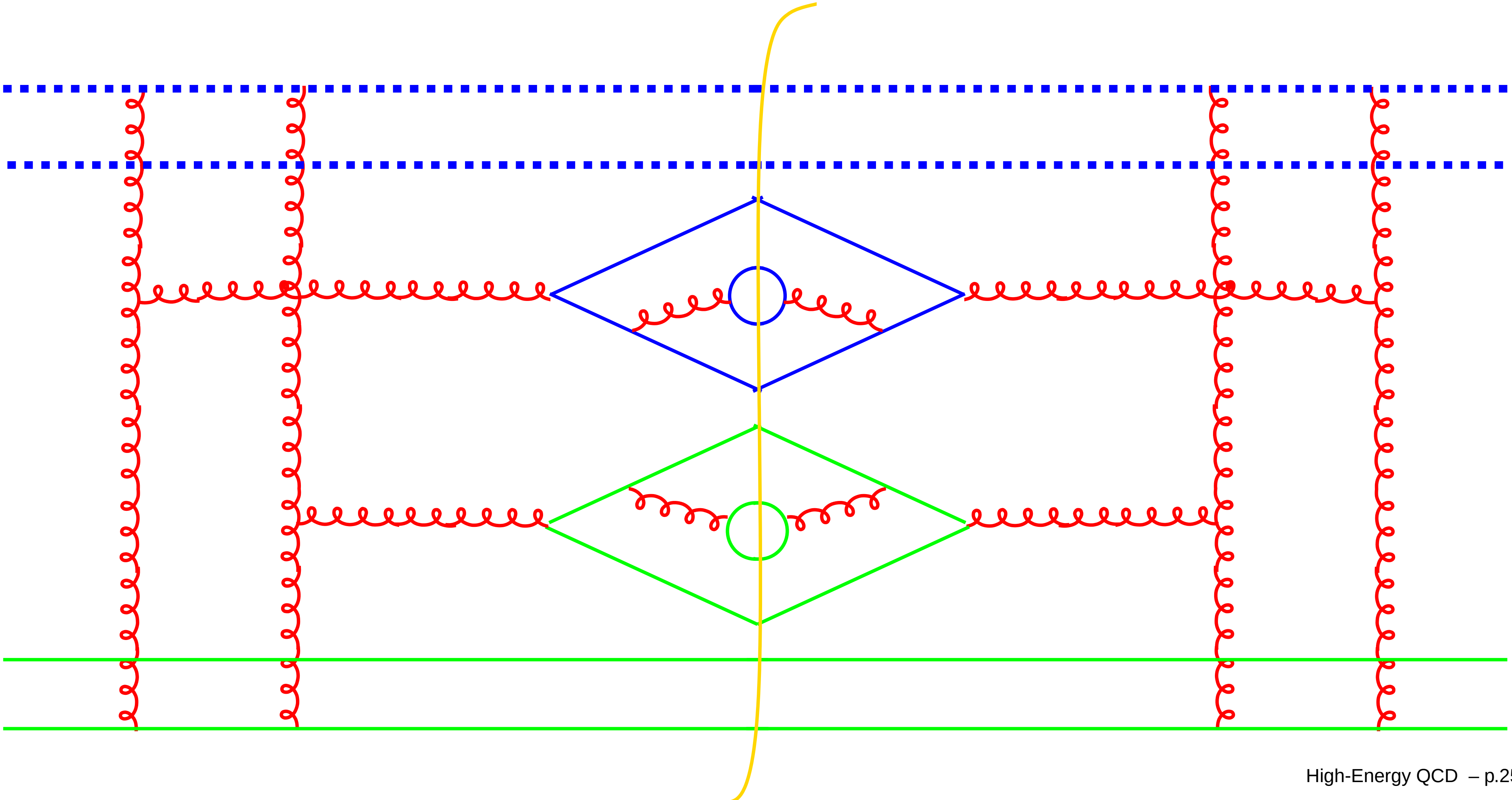
2 Wilson lines: propagation of the ‘‘color dipole’’



Wilson lines and color dipoles

$$D_g(x_B; x_\perp^{-2}) = \langle \text{meson} | \text{Tr}\{U^\eta(x_\perp) U^{\dagger\eta}(0)\} | \text{meson} \rangle$$

$\eta \equiv \ln \frac{1+v/c}{1-v/c} = \ln x_B$ - rapidity



Non-linear evolution of the color dipole

Color dipole: $\mathcal{U}^\eta(x_\perp, y_\perp) \equiv 1 - \frac{1}{3}\mathrm{Tr}\{U_x^\eta U_y^{\dagger\eta}\}$

Evolution equation ($x \equiv x_\perp$, $y \equiv y_\perp$)

$$\frac{d}{d\eta}\mathcal{U}^\eta(x,y) = \frac{3\alpha_s}{4\pi^2}\int d^2z\, \frac{(x-y)^2}{(x-z)^2(y-z)^2}$$
$$\times[\underbrace{\mathcal{U}^\eta(x,z) + \mathcal{U}^\eta(y,z) - \mathcal{U}^\eta(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{U}^\eta(x,z)\mathcal{U}^\eta(z,y)}_{\text{BK}}]$$

Balitsky,1996
Kovchegov, 1999

2 trivial solutions: $\mathcal{U} = 0$ - no gluons, $\mathcal{U} = 1$ - black disc

Non-linear evolution of the color dipole

Color dipole: $\mathcal{U}^\eta(x_\perp, y_\perp) \equiv 1 - \frac{1}{3}\mathrm{Tr}\{U^\eta_x U^{\dagger\eta}_y\}$

Evolution equation ($x \equiv x_\perp$, $y \equiv y_\perp$)

$$\frac{d}{d\eta}\mathcal{U}^\eta(x,y) = \frac{3\alpha_s}{4\pi^2}\int d^2z \, \frac{(x-y)^2}{(x-z)^2(y-z)^2}$$
$$\times[\underbrace{\mathcal{U}^\eta(x,z) + \mathcal{U}^\eta(y,z) - \mathcal{U}^\eta(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{U}^\eta(x,z)\mathcal{U}^\eta(z,y)}_{\text{BK}}]$$

Balitsky,1996
Kovchegov, 1999

2 trivial solutions: $\mathcal{U} = 0$ - no gluons, $\mathcal{U} = 1$ - black disc

BFKL - LLA for DIS in pQCD

BK - LLA for DIS in sQCD

sQCD: $\alpha_s \ll 1, \alpha_s F_{\mu\nu} \sim 1$

Non-linear evolution of the color dipole

Color dipole: $\mathcal{U}^\eta(x_\perp, y_\perp) \equiv 1 - \frac{1}{3}\mathrm{Tr}\{U_x^\eta U_y^{\dagger\eta}\}$

Evolution equation ($x \equiv x_\perp$, $y \equiv y_\perp$)

$$\frac{d}{d\eta}\mathcal{U}^\eta(x,y) = \frac{3\alpha_s}{4\pi^2}\int d^2z \, \frac{(x-y)^2}{(x-z)^2(y-z)^2}$$
$$\times[\underbrace{\mathcal{U}^\eta(x,z) + \mathcal{U}^\eta(y,z) - \mathcal{U}^\eta(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{U}^\eta(x,z)\mathcal{U}^\eta(z,y)}_{\text{BK}}]$$

Balitsky,1996
Kovchegov, 1999

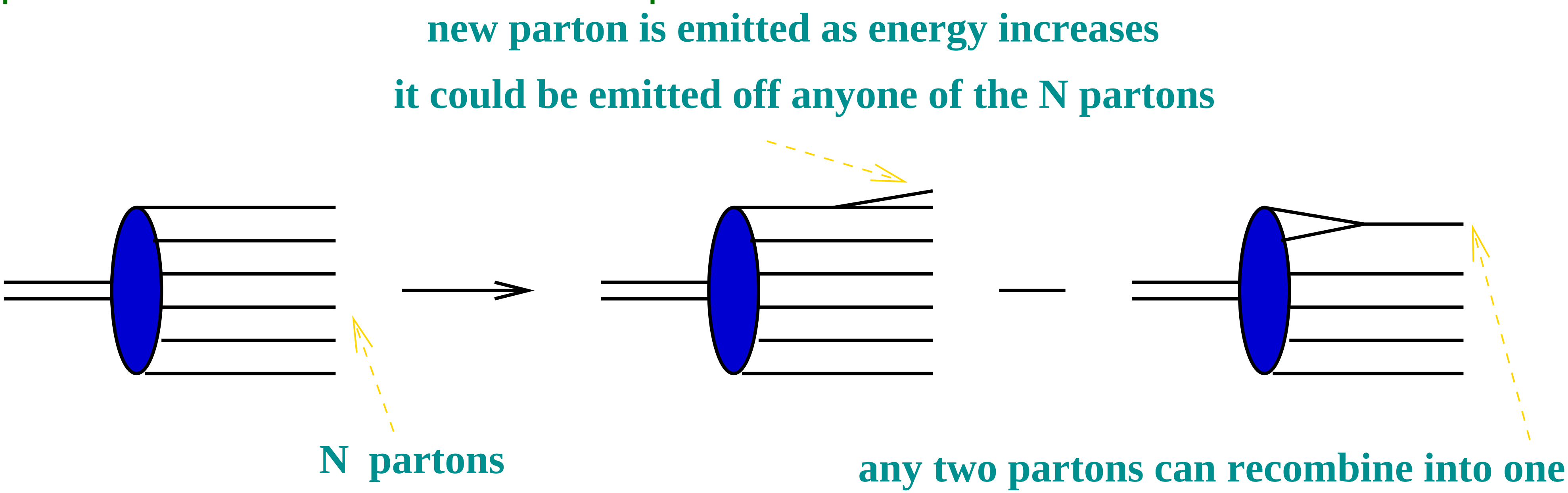
2 trivial solutions: $\mathcal{U} = 0$ - no gluons, $\mathcal{U} = 1$ - black disc

BFKL - LLA for DIS in pQCD $\alpha_s \ll 1, \alpha_s \ln x_B \sim 1$

BK - LLA for DIS in sQCD $\alpha_s \ll 1, \alpha_s \ln x_B \sim 1, \alpha_s A^{1/6} \sim 1$

sQCD: $\alpha_s \ll 1, \alpha_s F_{\mu\nu} \sim 1$ (example: large nuclei with $A \gg 1, \alpha_s A^{1/6} \sim 1$)

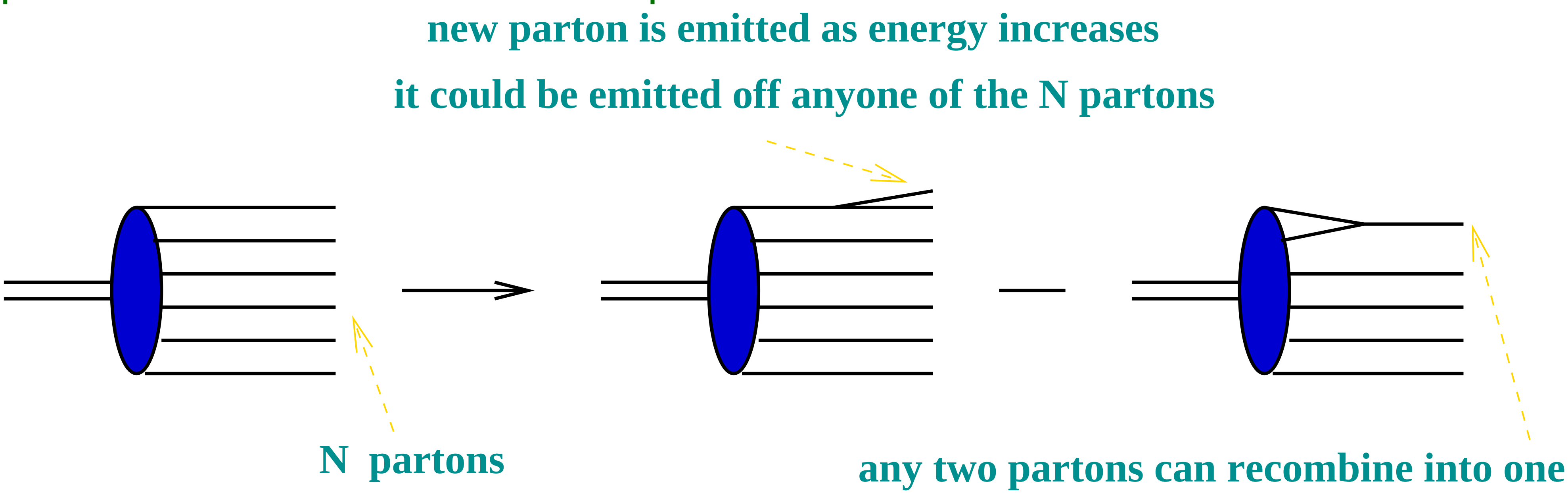
As we increase the energy, a new parton can be emitted from any of the N partons. The number of emitted particles $\sim N$.



At high energies, parton recombination becomes important.

$$\frac{dN(x_B, k_\perp^2)}{d\ln x_B} = \alpha_s K_{\mathrm{BFKL}} N(x_B, k_\perp^2) - \alpha_s [N(x_B, k_\perp^2)]^2$$

As we increase the energy, a new parton can be emitted from any of the N partons. The number of emitted particles $\sim N$.



At high energies, parton recombination becomes important.

$$\frac{dN(x_B, k_\perp^2)}{d\ln x_B} = \alpha_s K_{\mathrm{BFKL}} N(x_B, k_\perp^2) - \alpha_s [N(x_B, k_\perp^2)]^2$$

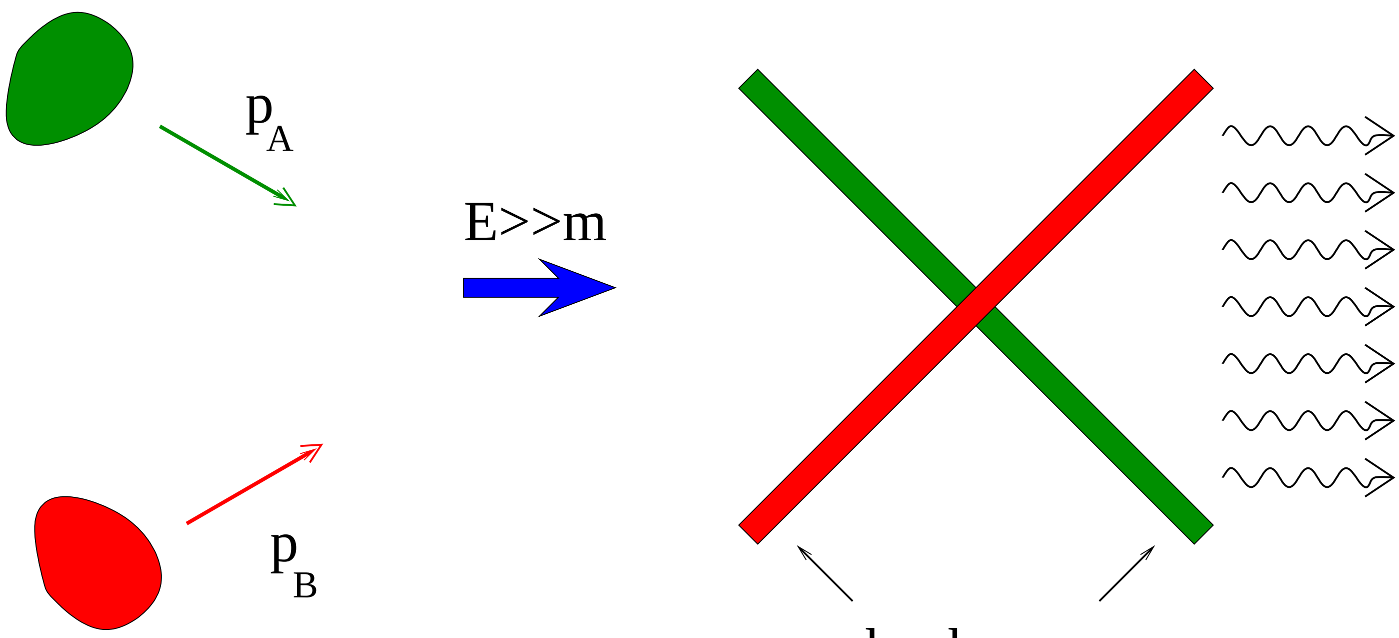
Eventually, recombination balances emission $\Rightarrow$ saturation

$k_\perp^{\mathrm{char}} \sim Q_s$ - saturation scale

Saturation is an old idea (Gribov,Levin,Ryskin, 1983),
but it has a lot of new development due to RHIC, heavy ions @ LHC, and future EIC accelerators.

Theoretical estimates: $Q_s \sim A^{1/3} e^{\lambda \alpha_s \eta} \Rightarrow$
$Q_s^2 = 1 \div 2$ GeV^2 for RHIC and $Q_s^2 = 2 \div 3$ GeV^2 for LHC
$\Rightarrow \alpha_s(Q_s)$ is small and one can apply ~~pQCD~~ sQCD

Gluon (semi)classical cloud of an energetic hadron in a state of saturation is called Color Glass Condensate (McLerran et al, 2001)



Scattering of two CGC's gives the initial conditions for the formation of a Quark-Gluon Plasma

A search for the 2+1 effective theory

Wanted:

$$\langle e^{i\rho^A iU(\eta_A)} e^{i\rho^B U(\eta_B)} \rangle_{\rm QCD} \stackrel{s\to\infty}{\longrightarrow}$$

$$\int \mathcal{D}U(z_\perp,\eta)\; e^{i\rho^A Ui(\eta_A)} e^{i\rho^B U(\eta_B)} \exp\left\{ i\int_{\eta_B}^{\eta_A} d\eta \int d^2z_\perp \mathcal{L}(U) \right\}$$

$\rho^A(z_\perp)$ and $\rho^B(z_\perp)$ - sources for the Wilson-line operators U

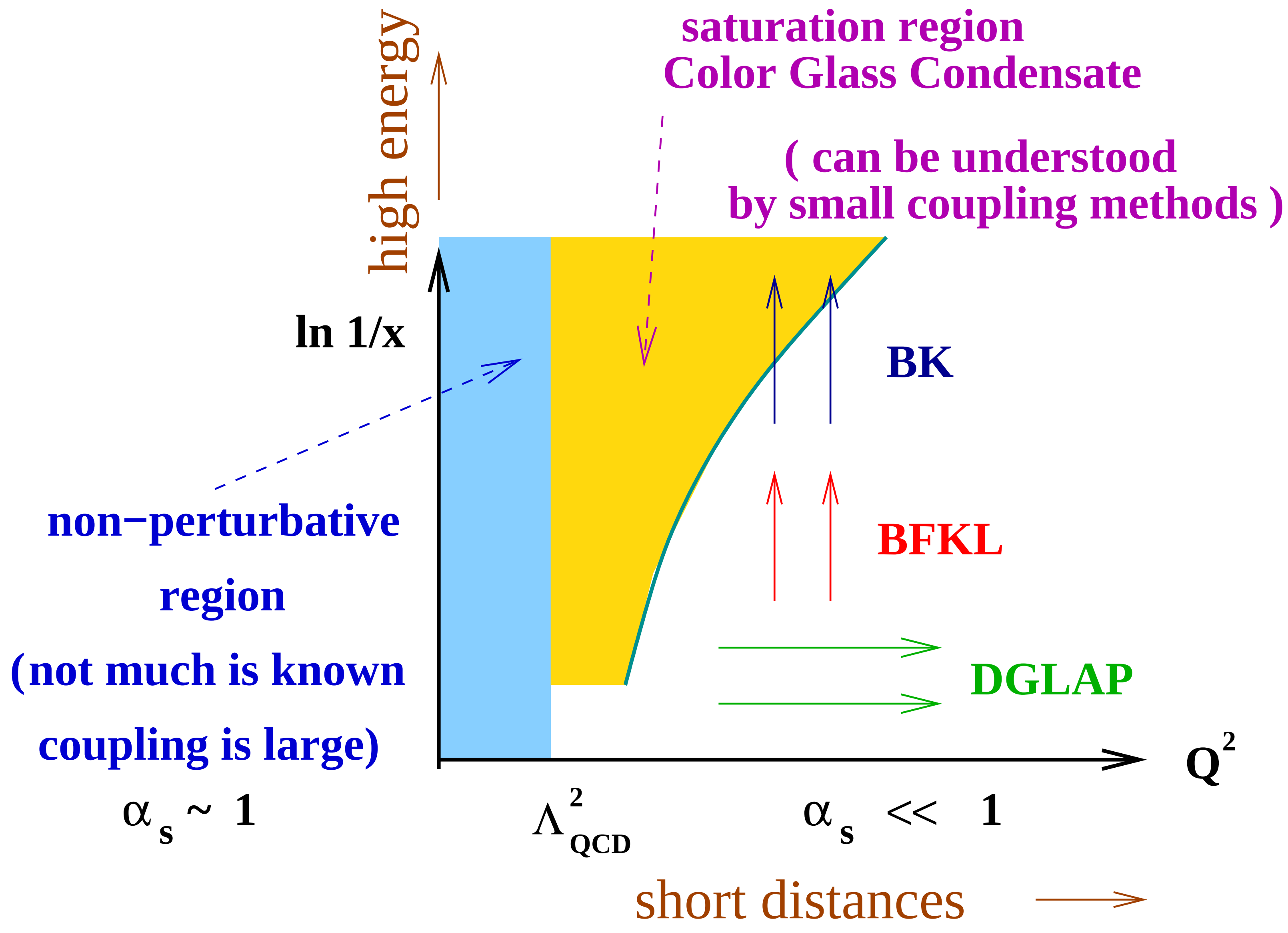
How to approach this goal?

- pQCD
- sQCD (s for semiclassical)
- symmetries?

Some progress has been made in recent year (McLerran, Stasto, Iancu, Hatta, Triantafallopoulos; Mueller, Shoshi; Kovner, Lublinsky; Balitsky; all in 2005)

but we do not have the simple expression for $\mathcal{L}$ yet.

Phase diagram of high-energy QCD



Conclusions

- High-energy scattering in the LLA is described by the BFKL equation.
- BFKL pomeron is a pre-asymptotic behavior at not very large energies
- Non-linear evolution at very large energies leads to the saturation of partons
- Saturation scale grows with energy $\longrightarrow$ sQCD for the heavy-ion collisions

Conclusions

- High-energy scattering in the LLA is described by the BFKL equation.
- BFKL pomeron is a pre-asymptotic behavior at not very large energies
- Non-linear evolution at very large energies leads to the saturation of partons
- Saturation scale grows with energy $\longrightarrow$ sQCD for the heavy-ion collisions

Outlook

- Froissart bound in QCD
- Effective 2+1 theory
- Initial conditions for the QGP