Problem 1.

A spaceship moves from Earth with speed $0.8c$. It emits light signals to Earth every 4s (in spaceship's frame). What are the time intervals between signals received at Earth?

Solution

Compare four events:

"A "- emission of a signal from the spaceship at time t_A and position x_A,

"B" - emission of a signal from the spaceship at time $t_B = t_A + \gamma(\Delta t')$ and position $x = x_A + v(t_B - t_A) = x_A + v\gamma(\Delta t')$,

"C" - the signal from "A" reaches Earth $\Rightarrow x_C = 0$, $t_C = t_A + \frac{x_A}{c}$, and

"D" - the signal from "B" reaches Earth $\Rightarrow x_D = 0,\ t_D = t_B + \frac{x_B}{c}$.

The time interval between signal received on Earth is therefore

$$t_D - t_C \;=\; t_B - t_A + \frac{x_B - x_A}{c} \;=\; \gamma(\Delta t') + \frac{v}{c}\gamma(\Delta t') \;=\; (\Delta t')\sqrt{\frac{c+v}{c-v}} \;=\; 3\Delta t' \;=\; 12\text{s}$$

Problem 2.

A π^0 meson with total energy 395 MEV (in the lab frame) decays into two photons in a symmetric way such that the energies of two photons are equal. Find the angle between photon's momenta.

Relevant information: a π^0 meson is an electrically neutral particle which can decay in two photons. The rest mass of the π^0 meson is $\frac{135\text{MeV}}{c^2}$.

Solution

Let the initial π^0 fly along x axis and two decay photons fly in x, y plane. Conservation of momentum is $p_x = k_x + k'_x = 2k_x$, $k_y = -k'_y$. Conservation of energy:

$$E_\pi = \sqrt{m_0^2c^4 + p_x^2c^2} = 2E_\gamma \;=\; 2c\sqrt{k_x^2 + k_y^2}$$

Since $p_x = 2k_x$

$$m_0^2c^4 + 4k_x^2c^2 = 2E_\gamma \;=\; 4k_x^2c^2 + 4k_y^2c^2 \quad\Rightarrow\quad k_y = \frac{m_0c}{2}$$

$$\rightarrow \quad \sin\frac{\theta}{2} \;=\; \frac{k_y}{\sqrt{k_x^2 + k_y^2}} \;=\; \frac{k_yc}{E_\pi/2} \;=\; \frac{m_0c^2}{E_\pi} \;=\; \frac{135}{395} \;\simeq\; 0.342$$

$\Rightarrow\ \frac{\theta}{2} \simeq 20^\circ$ so $\theta \simeq 40^\circ$

Alternatively,

$$\left.\begin{array}{lcl} k_x = \frac{p_x}{2} & = & \frac{1}{2c}\sqrt{E_\pi^2 - m_0^2c^4} \\ \sqrt{k_x^2+k_y^2} & = & \frac{E_\gamma}{c} \;=\; \frac{E_\pi}{2c} \end{array}\right\} \quad\Rightarrow\quad \cos\frac{\theta}{2} \;=\; \frac{\sqrt{E_\pi^2 - m_0^2c^4}}{E_\pi} \;=\; 0.94 \quad\Rightarrow\quad \frac{\theta}{2} \;=\; 20^\circ$$

One more solution

$$\left.\begin{array}{l} p_\pi^2 \;=\; \frac{E_\pi^2 - m_0^2c^4}{c^2} \;=\; (\vec{k}+\vec{k}')^2 \;=\; 2k^2 + 2k^2\cos\theta \\ E_\pi = 2E_\gamma \;\Rightarrow\; E_\gamma = kc = \frac{E_\pi}{2} \end{array}\right\} \quad\Rightarrow\quad \cos\theta \;=\; \frac{p_\pi^2}{2k^2} - 1 \;=\; 1 - 2\frac{m_0^2c^4}{E_\pi^2} \;=\; 0.766s$$

Problem 3.

An electromagnetic wave with electric field with time dependence

$$E \;=\; E_0(1+\cos\omega t)\cos\omega_0 t$$

is incident on lithium metal ($\phi = 2.39\text{eV}$). If $\omega = 1.8\times 10^{15}\frac{1}{\text{s}}$ and $\omega_0 = 2.4\times 10^{15}\frac{1}{\text{s}}$, what is the maximal kinetic energy of photoelectrons?

Solution

Since

$$(1+\cos\omega t)\cos\omega_0 t \;=\; \cos\omega t + \frac{1}{2}\cos(\omega+\omega_0)t + \frac{1}{2}\cos(\omega_0-\omega)t$$

$\Rightarrow$ the largest (angular) frequency in the wave is $\omega+\omega_0$

$$E_{\text{max}} \;=\; \hbar(\omega+\omega_0) - \phi \;=\; 6.58\times 10^{-16}\text{eV}\cdot\text{s}\times 4.2\times 10^{15}\frac{1}{\text{s}} - 2.39\text{eV} \;\simeq\; 0.37\text{eV}$$

Problem 4.

An electron in hydrogen atom moves from the first Bohr's orbit to the fourth one. What is the change of its potential energy according to Bohr's model?

Change of total energy is

$$\Delta E \;=\; E_0\Big(1-\frac{1}{16}\Big) \;=\; \frac{15}{16}\times 13.6\text{eV} \;=\; 12.75\text{eV} \;\simeq\; 2.04\times 10^{-18}\text{J}$$

The change in potential energy is twice that big:

$$\Delta E_{\text{pot}} \;=\; 2E_0\Big(1-\frac{1}{16}\Big) \;=\; \frac{15}{8}\times 13.6\text{eV} \;=\; 25.5\text{eV} \;\simeq\; 4.1\times 10^{-18}\text{J}$$