Problem.

In a certain frame K the electric field $\vec{E}$ and the magnetic filed $\vec{B}$ are orthogonal. Is there a frame where the field is
(a) purely electric or (b) purely magnetic,
and with what velocity should that frame(s) move with respect to K?

Solution

In the frame K the second Lorentz invariant $\vec{E}\cdot\vec{B}=0$ while the first invariant E^2-B^2 is positive when $|\vec{E}|>|\vec{B}|$ and negative for $|\vec{E}|<|\vec{B}|$ so nothing forbids to have $\vec{B}'=0$ in the former case and $\vec{E}'=0$ in the latter. To get the velocity of the relevant boost, consider the Lorentz transformations of the electric and magnetic fields

$$\begin{aligned}\vec{E}' &= \gamma(\vec{E}+\vec{\beta}\times\vec{B})-\frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{E})\\ \vec{B}' &= \gamma(\vec{B}-\vec{\beta}\times\vec{E})-\frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B})\end{aligned}$$

In the first case (when $E>B$) we want to have $B'=0$ so

$$(\vec{B}-\vec{\beta}\times\vec{E}) = \frac{\gamma}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B})$$

Multiplying both sides by $\vec{\beta}$ we get

$$\vec{\beta}\cdot(\vec{B}-\vec{\beta}\times\vec{E}) = \frac{\gamma}{\gamma+1}\beta^2(\vec{\beta}\cdot\vec{B}) \;\Rightarrow\; \vec{\beta}\cdot\vec{B}=0 \;\Leftrightarrow\; \vec{v}\perp\vec{B}$$

so $\vec{B}=-\vec{\beta}\times\vec{E}$. The simplest choice is to take $\vec{v}$ orthogonal to both $\vec{E}$ and $\vec{B}$, then

$$v=c\frac{B}{E},\quad \vec{v}\perp\vec{E},\vec{B}$$

Similarly, in the second case we can take $v=c\frac{E}{B},\;\; \vec{v}\perp\vec{E},\vec{B}$ and the resulting $\vec{E}'$ will vanish.

(In the SI units $v=c^2\frac{B}{E}$ and $v=\frac{E}{B}$ for the first and the second case, respectively).