A rigid uniform bar of mass m, weight mg, and length l is supported in equilibrium in a horizontal position by two massless springs attached one at each end. The springs have the same force constants k. The center-of-mass is constrained to move parallel to the vertical z-axis, and the bar oscillates in the xz-plane. Note that the moment of inertia of the bar with respect to the center-of-mass is $ml^2/12$.

1. How many degrees of freedom does the system have ?
2. Choose a set of generalized coordinates and obtain the Lagrangian.
3. Find the normal modes and frequencies of vibration of the system.