A railroad car is moving along a straight line in x-direction, with velocity $v(t)$. Using coordinates (x, y, z) relative to one corner of the car, write down the Lagrangian $\mathcal{L}$ for a particle of mass m moving around in this car without friction (make sure you calculate the kinetic and potential energies in the "fixed" ground reference frame). Calculate all generalized momenta p_x, p_y, p_z and then convert the Lagrangian to the corresponding Hamiltonian via the Legendre transformation

$$H(q_i, p_i) = \Sigma_i p_i \dot{q}_i - \mathcal{L} \tag{1}$$

Answer the following questions:

1. Does the *value* of the Hamiltonian at any given time equal the total mechanical energy (kinetic plus potential) of the system? If not, what is the difference?
2. Under what circumstances is the Hamiltonian a conserved quantity (i.e., its value is independent of time)?
3. For each of the three momentum components, state if (and under what condition) they are conserved.
4. Under what circumstances is the total mechanical energy of this system conserved? How does this relate to your answers to questions 1-3?

Justify your answers!