A puck of mass m is moving without friction on the $x - y$ plane and is attached to the origin ($x = y = 0$) with a spring of spring constant k and a "relaxed" length L.

1. Using polar coordinates r, ϕ, write down the Lagrangian for this situation
2. Write down the generalized momenta and determine which ones are conserved.
3. Write down the Euler-Lagrange equations for the coordinates.
4. Find the necessary condition for equilibrium motion at fixed distance r_0 around the center with constant angular velocity $\dot{\phi} = \omega$, i.e, express r_0 in terms of ω.
5. Parametrize small deviations from the equilibrium by introducing new coordinates δr and $\delta\phi$ with $r(t) = r_0 + \delta r(t)$ and $\phi(t) = \omega t + \delta\phi(t)$. In the following, assume that $\delta\phi(t = 0) = \delta\dot{\phi}(t = 0) = \delta\dot{r}(t = 0) = 0$ while $\delta r(t = 0)$ is not necessarily zero. Plug this ansatz into your Euler-Lagrange equations and evaluate by discarding all terms that contain more than one small factor (δr or $\delta\phi$ and their derivatives), as well as using the equilibrium condition to replace r_0.
6. Show that the resulting equations have as their solutions small oscillations of δr around the equilibrium value and calculate the frequency of these oscillations in terms of ω and L. (Don't worry about the corresponding solution for $\delta\phi$.)