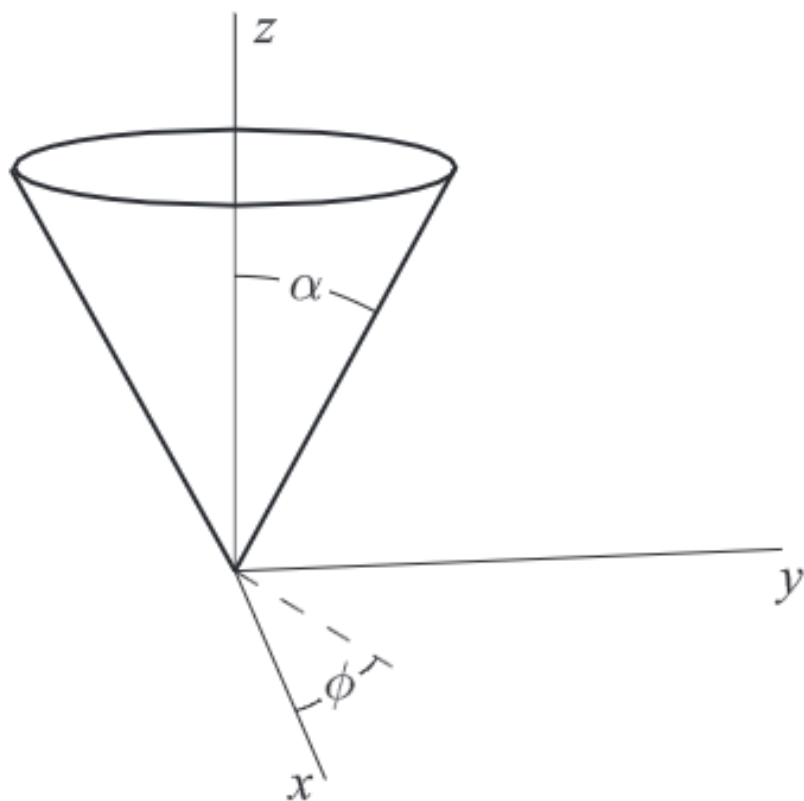Consider a particle which is subject to gravity but constrained to move on the surface of a cone whose apex is at the origin ($x = y = z = 0$) and whose opening angle is 2α (see figure).

(a) Write the Lagrangian for the particle in cylindrical coordinates r, ϕ.

(b) Derive the equations of motion.

(c) Find two conserved quantities related to the motion of the particle.