

Solution

The capacitance of two concentric cylinders per unit length is

$$C = \frac{2\pi\epsilon_0}{\ln b/a}$$

(see HW3 - problem 1.6c from Jackson). For a capacitor filled with dielectric $\epsilon_0 \rightarrow \epsilon$ so the capacitance of the setup is

$$C = \frac{2\pi\epsilon_0}{\ln b/a} [L - h + h \frac{\epsilon}{\epsilon_0}] = \frac{2\pi\epsilon_0}{\ln b/a} [L + h\chi_e]$$

and therefore $W = \frac{\pi\epsilon_0}{\ln b/a} [L + h\chi_e] V^2$ The force acting on dielectric may be found from

$$F = \left(\frac{\partial W}{\partial h} \right)_V = \frac{\pi\epsilon_0\chi_e}{\ln b/a} V^2$$

In the equilibrium position this force is balanced by weight $mg = \rho gh\pi(b^2 - a^2)$ so we get

$$h = \frac{\epsilon_0\chi_e V^2}{\rho g(b^2 - a^2) \ln b/a}$$