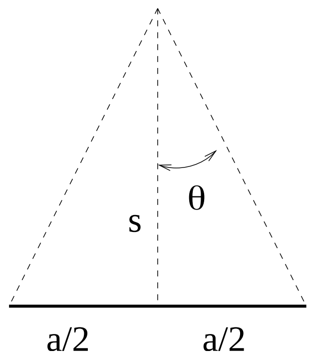
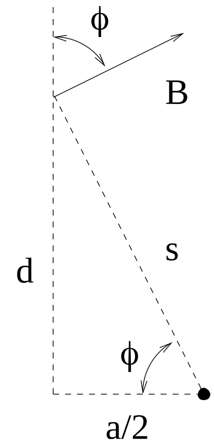
Solution

Let us start with the calculation of the magnetic field.





For one segment of the wire

$$\vec{B} \;=\; \frac{\mu_0 I}{4\pi}\int_{-a/2}^{a/2} dl \frac{\hat{l}\times(l\hat{l}+s\hat{s})}{(l^2+s^2)^{3/2}} \;=\; \frac{\mu_0 I}{4\pi}\hat{l}\times\hat{s}\int_{-a/2}^{a/2} dl \frac{s}{(l^2+s^2)^{3/2}}$$

$$\int_{-a/2}^{a/2} dl \frac{1}{(l^2+s^2)^{3/2}} \;\overset{l=s\tan\alpha}{=}\; \frac{1}{s^2}\int_{-\theta}^{\theta} d\alpha\;\cos\alpha \;=\; \frac{2\sin\theta}{s^2} \;=\; \frac{a}{s^2\sqrt{s^2+\frac{a^2}{4}}}$$

$$\Rightarrow\; B \;=\; |\vec{B}| \;=\; \frac{\mu_0 I a}{4\pi s\sqrt{s^2+a^2}} \tag{1}$$

By symmetry, the sum of contributions due to the four segments is pointing in z direction (orthogonal to the square). The z-component of Eq. 1 is

$$B_z = B\cos\phi = \frac{\mu_0 a^2 I}{8\pi s^2\sqrt{d^2+\frac{a^2}{2}}} \;=\; \frac{\mu_0 a^2 I}{8\pi(d^2+\frac{a^2}{4})\sqrt{d^2+\frac{a^2}{2}}}$$

and therefore

$$\vec{B} = 4B_z\hat{e}_3 = \frac{\mu_0 a^2 I\hat{e}_3}{2\pi(d^2+\frac{a^2}{4})\sqrt{d^2+\frac{a^2}{2}}}$$

As to the magnetic vector potential, it vanishes at $x, y = 0$ by symmetry.