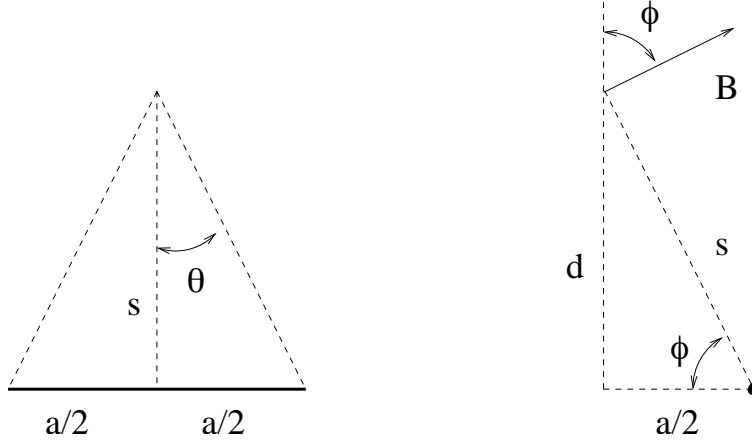


Solution

Let us start with the calculation of the magnetic field.



For one segment of the wire

$$\begin{aligned}\vec{B} &= \frac{\mu_0 I}{4\pi} \int_{-a/2}^{a/2} dl \frac{\hat{l} \times (\hat{l} + s\hat{s})}{(l^2 + s^2)^{3/2}} = \frac{\mu_0 I}{4\pi} \hat{l} \times \hat{s} \int_{-a/2}^{a/2} dl \frac{s}{(l^2 + s^2)^{3/2}} \\ &\int_{-a/2}^{a/2} dl \frac{1}{(l^2 + s^2)^{3/2}} \stackrel{l=s \tan \alpha}{=} \frac{1}{s^2} \int_{-\theta}^{\theta} d\alpha \cos \alpha = \frac{2 \sin \theta}{s^2} = \frac{a}{s^2 \sqrt{s^2 + \frac{a^2}{4}}} \\ \Rightarrow B &= |\vec{B}| = \frac{\mu_0 I a}{4\pi s \sqrt{s^2 + a^2}}\end{aligned}\quad (1)$$

By symmetry, the sum of contributions due to the four segments is pointing in z direction (orthogonal to the square). The z -component of Eq. 1 is

$$B_z = B \cos \phi = \frac{\mu_0 a^2 I}{8\pi s^2 \sqrt{d^2 + \frac{a^2}{2}}} = \frac{\mu_0 a^2 I}{8\pi (d^2 + \frac{a^2}{4}) \sqrt{d^2 + \frac{a^2}{2}}}$$

and therefore

$$\vec{B} = 4B_z \hat{e}_3 = \frac{\mu_0 a^2 I \hat{e}_3}{2\pi (d^2 + \frac{a^2}{4}) \sqrt{d^2 + \frac{a^2}{2}}}$$

As to the magnetic vector potential, it vanishes at $x, y = 0$ by symmetry.