

**Problem 1**

Find the magnetic moment of the uniformly charged spherical shell (charge  $Q$ , radius  $R$ ) spinning with angular velocity  $\omega$  about the  $z$  axis.

**Solution**

The surface current density for a spinning spherical shell is

$$\vec{K} = \sigma \vec{v} = \sigma \vec{\omega} \times \vec{r} = \sigma \omega R \sin \theta \hat{\theta}$$

A thin strip of the shell corresponding to angles between  $\theta$  and  $\theta + d\theta$  can be approximated by a (circle) wire carrying the current  $dI = K dl = KR d\theta$ . The radius of the “wire” is  $R \sin \theta$  and therefore the magnetic moment coming from this strip is

$$dI \pi (R \sin \theta)^2 \hat{e}_3 = KR^3 \sin^2 \theta d\theta \hat{e}_3 = \sigma \omega R^4 \sin^3 \theta d\theta \hat{e}_3$$

The total magnetic moment is

$$\vec{m} = \hat{e}_3 \int_0^\pi \sigma \omega R^4 \sin^3 \theta d\theta \hat{e}_3 = \frac{4\pi}{3} \sigma \omega R^4 = \frac{1}{3} Q \omega R^2$$

**Problem 2** (Pr. 5.55 from Griffiths)

A magnetic dipole  $\vec{m} = -m_0 \hat{e}_3$  is located at the origin, in an otherwise uniform magnetic field  $\vec{B} = B_0 \hat{e}_3$ . Show that there exist a spherical surface (centered at the origin) through which no magnetic field lines pass and find the radius of this surface.

**Solution**

Magnetic field lines are collinear to  $\vec{B}$  therefore if no magnetic lines pass through the spherical surface then  $B_r = 0$ .

The magnetic field of our setup (in spherical polar coordinates) has the form

$$\vec{B} = B_0 \hat{e}_3 - \frac{\mu_0 m_0}{4\pi r^3} (2\hat{r} \cos \theta + \hat{\theta} \sin \theta)$$

Since  $\hat{e}_3 = \hat{r} \cos \theta - \hat{\theta} \sin \theta$  we get

$$\vec{B} = \hat{r} \left( B_0 - \frac{\mu_0 m_0}{2\pi r^3} \right) \cos \theta - \hat{\theta} \left( B_0 + \frac{\mu_0 m_0}{4\pi r^3} \right) \sin \theta$$

We see that at

$$r = \left( \frac{\mu_0 m_0}{2\pi B_0} \right)^{1/3}$$

$B_r = 0$  (at any  $\theta$  and  $\phi$ ) and therefore no field lines cross this spherical surface.