a): By symmetry, the solution must be of the form $\rho(\mathbf{x}) = \rho(r) = Q\delta(r-R)f$, with a constant f to be specified by the condition

$$Q = \int \rho(r) 4\pi r^2 dr = \int Q\delta(r-R) f 4\pi r^2 dr = Qf4\pi R^2 \,. \tag{1}$$

Thus, $\boxed{\rho(r) = \frac{Q\delta(r-R)}{4\pi R^2}}$.

b): By symmetry, the solution must be of the form $\rho(\mathbf{x}) = \rho(r) = \lambda\delta(r-b)f$, with a constant f to be specified by the condition

$$\lambda = \int \rho(r) 2\pi r dr = \int \lambda\delta(r-b) f 2\pi r dr = 2\lambda f \pi b \,. \tag{2}$$

Thus, $\boxed{\rho(r) = \frac{\lambda\delta(r-b)}{2\pi b}}$.

c): By symmetry, the solution must be of the form $\rho(\mathbf{x}) = \rho(r,z) = \delta(z)\Theta(R-r)f(r)$. There, Θ is the step function, and $f(r)$ a function specified by a normalization condition that describes how much charge is supposed to be on a ring with radius r and radial thickness dr (for $r < R$):

$$Q\frac{2\pi r dr}{\pi R^2} = \int_z \rho(r,z) 2\pi r dz dr = \int_z \delta(z) f(r) 2\pi r dz dr = f(r) 2\pi r dr \,. \tag{3}$$

Thus, $\boxed{\rho(r,z) = \frac{Q\delta(z)\Theta(R-r)}{\pi R^2}}$.

d): By symmetry, the solution must be of the form $\rho(\mathbf{x}) = \rho(r,\theta) = \delta(\cos\theta)\Theta(R-r)f(r)$, where $f(r)$ is a function specified by a normalization condition that describes how much charge is supposed to be on a shell with radius r and radial thickness dr (for $r < R$):

$$Q\frac{2\pi r dr}{\pi R^2} = \int_{\cos\theta,\phi} \rho(r,\theta) r^2 d\phi d\cos\theta \, dr = 2\pi \int_{\cos\theta} \delta(\cos\theta) f(r) r^2 d\cos\theta \, dr = 2\pi f(r) r^2 dr \,. \tag{4}$$

Thus, $\boxed{\rho(r,\cos\theta) = \frac{Q\delta(\cos\theta)\Theta(R-r)}{\pi R^2 r}}$.