804 Final Exam (40 points).

Problem 1.

Consider the vector potential $A(\vec{r}) = \vec{r} \times (\vec{r} \times \hat{e}_3)$.

(a) What is the magnetic field?

(b) Does this potential satisfy Coulomb gauge condition $\vec{\nabla} \cdot \vec{A} = 0$? If not, modify $\vec{A}$ such that the magnetic field remains the same but the Coulomb condition is satisfied.

Problem 2.

A TEM wave propagates along a coaxial cylindrical wave guide (made from a perfect conductor) with inner radius a and outer radius b. Find the surface current on the outer cylinder. Is there a total current flow along the outer cylinder?

(Reminder/hint: a cylindrically symmetric solution of the 2-dim Laplace eqn is $\vec{E} \sim \hat{s}/s$ for the electric field and $\vec{B} \sim \hat{\phi}/s$ for the magnetic field)

Problem 3 .

A particle of charge q moves in a circle of radius a at a constant angular velocity ω. Assume that the circle lies in the x, y plane, centered at the origin and at time $t = 0$, the charge is at $(a, 0)$ on the positive x axis. For points on the z axis, find

(a) the Lienard-Wiechert potentials and

(b) the time-averaged electric field.

Problem 4.

An insulated circular ring of radius b lies in the x, y plane centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \sin\phi$ where ϕ is an azimuthal angle. The ring is set spinning at a constant angular velocity $\vec{\omega} = \omega\hat{z}$. Find the power radiated by the ring.

Problem 5.

In a certain frame K the electric field $\vec{E}$ and the magnetic filed $\vec{B}$ are orthogonal. Is there a frame where the field is

(a) purely electric or (b) purely magnetic,

and with what velocity should that frame(s) move with respect to K?