Phys. 804 — Classical Electrodynamics

HW 2 Solution

(a) $f(\vec{x}', t') \;=\; \delta(x')\delta(y')\delta(t')$

$$\psi(\vec{x}, t) \;=\; \int d^3x' \;\frac{f(\vec{x}', t_{\rm ret})}{|\vec{x}-\vec{x}'|} \;=\; \int d^3x' \;\frac{f(\vec{x}', t-\frac{|\vec{x}-\vec{x}'|}{c})}{|\vec{x}-\vec{x}'|}$$

$$= \;\int dx'dy'dz' \;\frac{1}{|\vec{x}-\vec{x}'|}\delta(x')\delta(y')\delta(t-\frac{|\vec{x}-\vec{x}'|}{c})$$

$$= \;\int dz' \;\frac{1}{\sqrt{x^2+y^2+(z-z')^2}}\delta(t-\frac{\sqrt{x^2+y^2+(z-z')^2}}{c})$$

$$= \;\int dz' \;\frac{\delta(t-\frac{\sqrt{x^2+y^2+z'^2}}{c})}{\sqrt{x^2+y^2+z'^2}} \;=\; \frac{1}{t}\int dz' \;\delta(\sqrt{x^2+y^2+z'^2}-ct)$$

$$= \;\frac{1}{t}\int dz' \;\frac{\sqrt{x^2+y^2+z'^2}}{|z'|}[\delta(z'-\sqrt{c^2t^2-x^2-y^2})+\delta(z'-\sqrt{c^2t^2-x^2-y^2})]$$

$$= \;\frac{2c}{\sqrt{c^2t^2-x^2-y^2}}\theta(c^2t^2-x^2-y^2) \;=\; \frac{2c}{\sqrt{c^2t^2-\rho^2}}\theta(c^2t^2-\rho^2)$$

(b) $f(\vec{x}', t') \;=\; \delta(z')\delta(t')$

$$\psi(\vec{x}, t) \;=\; \int d^3x' \;\frac{f(\vec{x}', t_{\rm ret})}{|\vec{x}-\vec{x}'|} \;=\; \int d^3x' \;\frac{\delta(z')\delta(t-\frac{|\vec{x}-\vec{x}'|}{c})}{|\vec{x}-\vec{x}'|}$$

$$= \;\frac{1}{t}\int dx'dy' \;\delta(ct-\sqrt{z^2+(x-x')^2+(y-y')^2}) \;=\; \frac{1}{t}\int dx'dy' \;\delta(ct-\sqrt{z^2+x'^2+y'^2})$$

$$= \;\frac{1}{t}\int_0^\infty \rho d\rho\int_0^{2\pi} d\phi \;\delta(ct-\sqrt{z^2+\rho^2}) \;=\; \frac{\pi}{t}\int_0^\infty d\rho^2 \;\delta(\sqrt{z^2+\rho^2}-ct) \;=\; 2\pi c\;\theta(ct-\rho)$$