Problem

A linearly polarized electromagnetic plane wave is normally incident on an infinitely large plane made from a perfect conductor. Find the charge and current densities induced on the conducting plane.

Solution

Let us choose the z axis in the direction normal to the plane and the wave coming from above the plane with electric field polarized in x direction. The incident wave has the form

$$\vec{E} \ = \ \hat{e}_x E_0 e^{-i\omega t+ikz}, \quad \vec{B} \ = \ \frac{\hat{n}}{c} \times \vec{E} \ = \ -\frac{\hat{e}_z}{c} \times \vec{E} \ = \ -\hat{e}_y \frac{E_0}{c} e^{-i\omega t+ikz}$$

The reflected wave is

$$\vec{E} \ = \ \vec{E}_R e^{-i\omega t-ikz}, \quad \vec{B} \ = \ \frac{\hat{n}}{c} \times \vec{E} \ = \ \frac{\hat{e}_z}{c} \times \vec{E}_R \ e^{-i\omega t-ikz}$$

The boundary condition for the electric field is $E_{\parallel}^{\text{above}} = E_{\parallel}^{\text{below}} = 0$ so $\vec{E}_R = -\hat{e}_x E_0$ and the sum of the reflected and incident waves takes the form:

$$\vec{E} \ = \ 2i\hat{e}_x E_0 \sin kz \ e^{-i\omega t}, \quad \vec{B} \ = \ 2\hat{e}_y \frac{E_0}{c} \cos kz \ e^{-i\omega t}$$

Now, σ and K can be found from the remaining boundary conditions

$$\epsilon_1 \vec{E}_{\perp}^{\text{above}} - \epsilon_2 \vec{E}_{\perp}^{\text{below}} = \sigma, \quad \vec{K} = (\vec{H}^{\text{above}} - \vec{H}^{\text{below}}) \times \hat{n} \ = \ -\frac{1}{\mu_0} \hat{e}_z \times \vec{B}^{\text{above}}$$

We get $\sigma = 0$ and $\vec{K} \ = \ \frac{2}{\mu_0 c} \hat{e}_x E_0 \ e^{-i\omega t}$. Taking the real part we get

$$K_x \ = \frac{2}{\mu_0 c} E_0 \ \cos \omega t, \quad K_y \ = \ 0$$