Problems (from qual exams)

Problem 1.

Consider an electron (spin $\frac{1}{2}$) in a state which is an eigenstate of orbital angular momentum l = 1, an eigenstate of total angular momentum $j = \frac{1}{2}$, and with $j_z = \frac{1}{2}$.

Is it an eigenstate of L_z ? Find an expectation value of L_z .

Problem 2.

The spin part of a wave function of a system of spin $\frac{1}{2}$ particles has the form

$$\left(\begin{array}{c}1\\0\end{array}\right)_1\left(\begin{array}{c}1\\0\end{array}\right)_2\cdots\left(\begin{array}{c}1\\0\end{array}\right)_n\left(\begin{array}{c}0\\1\end{array}\right)_{n+1}\cdots\left(\begin{array}{c}0\\1\end{array}\right)_N$$

Find the average \hat{S}^2 in this state (S is the total spin of these particles).

Problem 3.

A beam of spineless nuclear particles of mass m and momentum p is directed along the z-axis. The particles collide with an aligned diatomic molecule but interact only with the nuclei of the molecule. If the nuclei are taken to be at y = b and y = -b, and the constant a is positive, the interaction potential is given by

$$V(r) = a\delta(y-b)\delta(x)\delta(z) + a\delta(y+b)\delta(x)\delta(z)$$

Calculate the scattering amplitude and the differential cross section in the Born approximation